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MENTAL ARITHMETIC

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THE

WILLIAMS & ROGERS

MENTAL ARITHMETIC

CONTAINING

THOROUGH ORAL DRILL

IN THE

PRINCIPLES OF ARITHMETIC

BY

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PREFACE.

This book is designed to revive the teaching of Mental Arithmetic by the use of oral analyses. The subject has been neglected for the past few years, many teachers thinking that the time given to it could be used to better advantage in other work, and that it could be included in the regular work of written arithmetic; but, after a trial without it, during which time it has been observed that pupils are doing less *thinking* and *reasoning* than formerly, there now seems to be a general desire among teachers to return to this method of teaching the subject as a means of securing more thought and greater accuracy in the work of arithmetic. When properly presented and thoroughly taught, it gives an impetus to the mind such as no other elementary branch of study incites.

No subject is better adapted to create interest and arouse enthusiasm in the class-room than that of mental arithmetic, and when in the hands of an *energetic* teacher, the results obtained in rapid and accurate computations are wonderful.

The book contains a thorough drill in the fundamental principles of arithmetic, and in Common Fractions, Decimal Fractions, Percentage, Interest, etc., etc., besides graded analyses of each subject. It is hoped that the forms of analysis here used will be the means of making clear what has seemed to many pupils the greatest difficulty in arithmetic.

All mathematical puzzles have been avoided, and only

such problems as are of practical use to pupils have been given.

Teachers should require that the introductory exercises which precede each principle be carefully read under their supervision, and, if necessary, explain parts to pupils until all are familiar with the principle and also with the terms used, which are here defined in order that they may have a full understanding of the subject and be better prepared to take up more advanced work. The definitions need not be committed to memory until each subject has been completed, if at all.

Review lessons are given in connection with each principle, including preceding principles, which require the pupil to determine the process and apply the analysis independently of the text-book. The cultivation of this independence is one of the chief aims of mental arithmetic.

The omission of a thorough course of this kind of mental work in a school deprives the pupil of one of the best means of gaining intellectual strength.

"Whatever cultivates care, close observation, patience and method, must be valuable training and preparation for all studies and all pursuits."

J. M. C.

Rochester, N. Y.

MENTAL ARITHMETIC.

ADDITION.

Introductory Exercises.

1. How many are 2 apples and 1 apple?

Answer. 2 apples and 1 apple are 3 apples.

2. How many are 2 books and 3 books?

3. How many are 4 and 2?

Ans. 4 and 2 are 6.

4. How many are 3 dollars and 4 dollars? 2 books and 5 books?

5. Can you unite 3 cents and 4 books? Why not?

6. Can you unite 3 cents and 5 cents? Why?

7. What kind of numbers can be united?

Like Numbers are those which express things of the *same kind.*

Thus, 2 apples and 3 apples, 7 pecks and 9 pecks, 4 and 5, 6 dollars and 8 dollars are like numbers.

Definitions.

1. Addition is the process of uniting two or more numbers of the same kind.

2. Sum or Amount is the number found by adding.

3. The Sign of Addition is the perpendicular cross, +, called *plus.* It indicates that the numbers connected by it are to be added.

Thus, 3 + 2 means that 3 and 2 are to be added, and is read 3 plus 2.

4. The SIGN of EQUALITY is two short horizontal parallel lines, =. It is read *equal to* or *equals.*

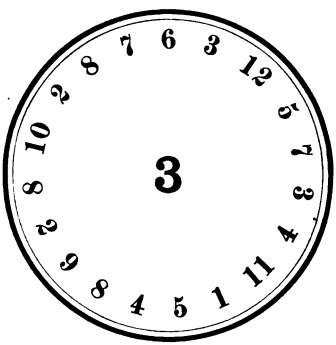
Thus, 4 + 3 = 7 is read 4 plus 3 equals 7, or 4 and 3 are 7, the Sum or Amount.

5. The SIGN ? is read *what* or *how many.*

Thus, 5 × 4 = ? is read 5 plus 4 equals what, or 5 plus 4 are how many.

NOTE TO TEACHER.—To awaken an interest in the recitation, to facilitate the memorizing of the different combinations, and for review, as well as to give variety to the exercises, the teacher will find the following method very useful with the younger pupils:

PRACTICE DIAGRAM.



The diagram should be placed upon the blackboard and allowed to remain where it may be easily seen by the pupils. The teacher should point to the numbers in the circle promiscuously, and in rapid succession, requiring the results *only* to be given.

Thus, the teacher points to 9, and some member of the class, designated by her, answers 12; to 4, and he responds 7. *Change the central number to correspond with each day's lesson.* To add interest to the recitation, teachers may occasionally have some *pupil* point to the numbers.

A few minutes of each day's recitation spent in exercises of this kind will soon make the pupil familiar with all the combinations.

Pupils may be required to add three numbers after the first few lessons have been learned or whenever the teacher sees that they are sufficiently advanced, *always combining with some number in a preceding lesson.*

Addition Table.

1 + 1 = 2	1 + 2 = 3	1 + 3 = 4	1 + 4 = 5
2 + 1 = 3	2 + 2 = 4	2 + 3 = 5	2 + 4 = 6
3 + 1 = 4	3 + 2 = 5	3 + 3 = 6	3 + 4 = 7
4 + 1 = 5	4 + 2 = 6	4 + 3 = 7	4 + 4 = 8
5 + 1 = 6	5 + 2 = 7	5 + 3 = 8	5 + 4 = 9
6 + 1 = 7	6 + 2 = 8	6 + 3 = 9	6 + 4 = 10
7 + 1 = 8	7 + 2 = 9	7 + 3 = 10	7 + 4 = 11
8 + 1 = 9	8 + 2 = 10	8 + 3 = 11	8 + 4 = 12
9 + 1 = 10	9 + 2 = 11	9 + 3 = 12	9 + 4 = 13
10 + 1 = 11	10 + 2 = 12	10 + 3 = 13	10 + 4 = 14
11 + 1 = 12	11 + 2 = 13	11 + 3 = 14	11 + 4 = 15
12 + 1 = 13	12 + 2 = 14	12 + 3 = 15	12 + 4 = 16
1 + 5 = 6	1 + 6 = 7	1 + 7 = 8	1 + 8 = 9
2 + 5 = 7	2 + 6 = 8	2 + 7 = 9	2 + 8 = 10
3 + 5 = 8	3 + 6 = 9	3 + 7 = 10	3 + 8 = 11
4 + 5 = 9	4 + 6 = 10	4 + 7 = 11	4 + 8 = 12
5 + 5 = 10	5 + 6 = 11	5 + 7 = 12	5 + 8 = 13
6 + 5 = 11	6 + 6 = 12	6 + 7 = 13	6 + 8 = 14
7 + 5 = 12	7 + 6 = 13	7 + 7 = 14	7 + 8 = 15
8 + 5 = 13	8 + 6 = 14	8 + 7 = 15	8 + 8 = 16
9 + 5 = 14	9 + 6 = 15	9 + 7 = 16	9 + 8 = 17
10 + 5 = 15	10 + 6 = 16	10 + 7 = 17	10 + 8 = 18
11 + 5 = 16	11 + 6 = 17	11 + 7 = 18	11 + 8 = 19
12 + 5 = 17	12 + 6 = 18	12 + 7 = 19	12 + 8 = 20
1 + 9 = 10	1 + 10 = 11	1 + 11 = 12	1 + 12 = 13
2 + 9 = 11	2 + 10 = 12	2 + 11 = 13	2 + 12 = 14
3 + 9 = 12	3 + 10 = 13	3 + 11 = 14	3 + 12 = 15
4 + 9 = 13	4 + 10 = 14	4 + 11 = 15	4 + 12 = 16
5 + 9 = 14	5 + 10 = 15	5 + 11 = 16	5 + 12 = 17
6 + 9 = 15	6 + 10 = 16	6 + 11 = 17	6 + 12 = 18
7 + 9 = 16	7 + 10 = 17	7 + 11 = 18	7 + 12 = 19
8 + 9 = 17	8 + 10 = 18	8 + 11 = 19	8 + 12 = 20
9 + 9 = 18	9 + 10 = 19	9 + 11 = 20	9 + 12 = 21
10 + 9 = 19	10 + 10 = 20	10 + 11 = 21	10 + 12 = 22
11 + 9 = 20	11 + 10 = 21	11 + 11 = 22	11 + 12 = 23
12 + 9 = 21	12 + 10 = 22	12 + 11 = 23	12 + 12 = 24

1.

Problems.

1. Fred has 6 peaches and George has 2; how many have both?

Solution.—They both have the sum of 6 peaches and 2 peaches which is 8 peaches. Therefore, they both have 8 peaches.

2. Two girls are studying and 5 are sewing; how many busy girls are there?

3. We planted 4 trees on Arbor Day this year and 2 last year; how many trees did we plant?

4. Ella saw 7 birds on one tree and 2 on another; how many birds did she see?

5. Frank earned 8 dollars and his younger brother earned 2; how many dollars did both earn?

6. Mr. Jones has 3 sons and 2 daughters; how many children has he?

7. Jane saw 3 electric cars on one street and 3 on another; how many cars did she see?

8. James gathered 3 quarts of nuts while Frank gathered 4 quarts; how many quarts did both gather?

9. Mr. Sill owns 3 gas stoves and Mr. Brown owns 5; how many do both own?

10. Mary worked 6 examples and Jennie 3; how many did both work?

11. Thomas had 7 bunches of fire-crackers and his brother gave him 3 bunches more; how many had he then?

12. I paid 5 cents for a slate and 3 cents for paper; how many cents did I spend?

13. John earned 6 dollars and Samuel earned 3; how many dollars did both earn?

14. Ned bought a lemon for 4 cents and an orange for 5 cents; how many cents did he pay for both?

15. Matilda bought tape for 6 cents and thread for 4 cents; how much did she pay for both?

16. Alfred had 7 rides on a donkey and 4 on a camel; how many rides had he?

17. Edna has 4 birds and 4 kittens; how many pets has she?

18. John had 3 marbles and found 4; how many marbles had he then?

19. Emma has 6 cents and Nora has 4; how many cents have both?

Practice Exercises.

Add, giving results only:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
20.	6	4	7	5	5	8	6	6	4	3	7	8
	2	4	3	3	4	2	4	3	3	4	4	3

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
21.	3	7	2	8	2	7	3	6	4	5	5	5
	2	2	3	3	4	4	3	4	2	3	4	2

22. A man sold 5 gallons of gasoline at one house and 5 at another; how many did he sell at both houses?

23. George has 5 light neckties and 4 dark ones; how many ties has he?

24. Sarah rode 6 miles one day and 5 miles the next; how many miles did she ride?

25. Charles had 5 rabbits and his father bought him 7 more; how many did he then have?

26. Grace made 7 presents and Nellie 5; how many presents did both make?

27. I saw 9 bicycles on one street and 5 on another; how many did I see on both streets?

28. Arthur had 3 bags of popcorn and George had 6; how many bags had both?

29. A man sold 6 books on Adams street and 4 on Tremont street; how many did he sell on both streets?

30. Adam sold 6 boxes of matches while Levi sold 5; how many boxes did both sell?

31. Cora and Jane have 6 rare stamps each; how many have they together?

32. In one pasture there are 6 cows and in another there are 7; how many cows in both pastures?

33. Freeman learned 10 new words on Monday and 6 on Tuesday; how many did he learn during both days?

34. Matilda spent 6 dollars and has 6 dollars left; how much had she at first?

35. I saw 6 crows in a field and 5 flying away; how many crows in all?

Practice Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
36.	6	3	5	4	8	2	3	4	3	9	5	6
	3	4	2	5	2	6	3	4	5	2	6	4
37.	6	7	5	5	6	7	10	9	6	8	3	7
	2	3	4	5	4	3	2	5	6	4	6	6

38. Flora's hat cost $3 and her dress $7; how much did both cost her?

Note.—The sign $ signifies dollars, and cts. or ¢, cents.

39. Laura had 5 pins in her cushion and 7 in her hand; how many pins had she?

40. How many are 6 and 5? 7 and 3? 4 and 3? 5 and 5? 4 and 9?

41. Cora had 6 apples and Alice gave her 7; how many apples had she then?

42. Mr. King spent $10 for a clock and $7 for a ring; how much money did he spend?

43. James earned $4, George $3 and Eddie $9; how many dollars did the three boys earn?

SOLUTION.—They earned the sum of $4, $3 and $9. $4 and $3 are $7 and $9 are $16. Therefore, the three boys earned $16.

44. I bought a book for $2, an umbrella for $5 and a satchel for $7; how much money did I expend?

NOTE.—Add, by use of the Practice Diagram, until results are perfectly familiar to pupils.

45. Add 2 to each of the following numbers: 3, 7, 9, 4, 1, 6, 2, 5, 8. Also add each of the following, 1, 6, 2, 5, 4, 7, 3, to the same numbers.

Give the results of the following:

46. 2 and 6 = ?	*47.* 2 plus 9 = ?	*48.* 6 + 8 = ?
7 and 3 = ?	3 plus 6 = ?	7 + 4 = ?
5 and 4 = ?	4 plus 8 = ?	5 + 7 = ?
4 and 4 = ?	5 plus 4 = ?	4 + 6 = ?
3 and 3 = ?	6 plus 7 = ?	3 + 8 = ?
2 and 2 = ?	7 plus 5 = ?	2 + 5 = ?
6 and 3 = ?	6 plus 7 = ?	7 + 6 = ?
4 and 9 = ?	4 plus 9 = ?	0 + 7 = ?
6 and 4 = ?	3 plus 8 = ?	6 + 9 = ?
3 and 9 = ?	2 plus 10 = ?	5 + 8 = ?

49. We had 5 picnics at Seneca Park and 8 at South Park; how many picnics had we?

50. Harry bought 8 pencils and Ned 5; how many did both buy?

51. Walter picked 8 quarts of berries and Reuben 7; how many quarts did both pick?

52. Nellie has 8 roses and 8 pansies; how many flowers has she?

53. Martha had $9 and earned $8 more; how many had she then?

54. Robert had 11 chickens and Ralph had 8; how many had both?

55. Kate bought 9 geraniums and 4 verbenas; how many plants did she buy?

56. If Helen spent 5 cents and her sister 9, how many did both spend?

57. Miss Young bought 12 bananas and Miss Elder bought 9; how many did both buy?

58. A boy saw 3 flocks of wild geese; if one contained 5 geese, another 4 and the third 8, how many geese were there in the three flocks?

59. After spending $10, I had $9 left; how many dollars had I at first?

60. Ned is now 9 years of age; how old will he be in 9 years?

61. 9 + 3 = ?	*62.* 2 plus 7 = ?	*63.* 2 and 2 = ?
3 + 4 = ?	3 plus 6 = ?	4 and 4 = ?
5 + 5 = ?	4 plus 5 = ?	6 and 6 = ?
4 + 6 = ?	5 plus 4 = ?	8 and 8 = ?
6 + 7 = ?	6 plus 3 = ?	1 and 1 = ?
2 + 8 = ?	7 plus 2 = ?	3 and 3 = ?
1 + 9 = ?	8 plus 1 = ?	5 and 5 = ?
0 + 5 = ?	9 plus 2 = ?	7 and 7 = ?
3 + 7 = ?	10 plus 3 = ?	9 and 9 = ?

64. After spending $5, I had $10 left; how much had I at first?

65. Mrs. Clifford spent $5 for a hat, $7 for shoes and $10 for a dress; how much did the three articles cost her?

66. There are 10 windows upon one side of our house and 7 upon the other; what is the whole number of windows upon both sides?

67. Flora hemmed 6 towels and Minnie 10; how many towels did both girls hem?

68. A farmer sold 9 bushels of potatoes to one man and 10 to another; how many bushels did he sell to both?

69. John paid 7 cents for a top and 10 cents for a ball; how much did he pay for both?

70. Mary gave me 11 pencils and Beula gave me 8; how many pencils did I receive from both?

71. Mr. Smith built 7 houses this year and 11 last year; how many houses did he build both years?

72. Carrie studied Botany 9 weeks and Chemistry 11 weeks; how many weeks did she spend in the study of both?

73. If Herbert is 11 years of age and his sister is 9, what is the sum of their ages?

74. I spent $11 for a cape and $4 for a pair of shoes; how much did I spend for both?

75. Mrs. Williams furnished 5 cakes for the supper at the Y. M. C. A., Mrs. Brewster 3, and Mrs. Brown 11; how many cakes did the three ladies furnish?

SOLUTION.—The three ladies furnished the sum of 5 cakes, 3 cakes and 11 cakes. 5 cakes and 3 cakes are 8 cakes, and 11 cakes are 19 cakes. Therefore, they all furnished 19 cakes.

76. If Harry was 12 years old 3 years ago, how old is he now?

77. Jane is now 8 years old; how old will she be in 12 years?

78. Mary had 12 Christmas presents and her small brother had 9; how many had both?

79. Yesterday we had 7 visitors at our school and to-day 12; how many visitors had we in all?

80. Mr. Otis earned 12 dollars and his son earned 3 dollars; how much did both earn?

81. Mr. Wright bought 6 books, Mr. King 4, and Mr. Bristol 12; how many books did the three men buy?

82. How many are 9 and 9? 8 and 8? 7 and 7? 6 and 6? 5 and 5?

83. What numbers added will give 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15?

84. Count by 2's to 30; by 3's to 30; by 4's to 28; by 5's to 30; by 6's to 30; by 7's to 42.

85. How many are 3, 4, 5? 6, 2, 3? 8, 1, 9? 1, 2, 3, 4? 6, 0, 8? 3, 7, 4? 9, 2, 11? 7, 6, 1?

86. How many are 4, 2, 9? 6, 5, 4? 3, 2, 7? 5, 4, 9? 7, 1, 6, 5?

87. How many are 1, 3, 2, 5? 9, 2, 7, 2? 12, 3, 4? 5, 6, 4, 3? 1, 2, 3, 4, 5?

88. How many are 6, 7, 3, 2? 1, 3, 5, 7, 4? 1, 5, 9, 2?

89.—	*90.*—	*91*—.
What is the sum of 8 and 5?	3 plus 7 = ?	6 and 5 = ?
What is the sum of 9 and 10?	11 plus 3 = ?	7 and 6 = ?
What is the sum of 4 and 7?	12 plus 5 = ?	1 and 7 = ?
What is the sum of 6 and 5?	2 plus 10 = ?	8 and 8 = ?
What is the sum of 7 and 6?	4 plus 5 = ?	9 and 9 = ?
What is the sum of 8 and 7?	8 plus 9 = ?	4 and 5 = ?
What is the sum of 9 and 8?	5 plus 7 = ?	10 and 4 = ?
What is the sum of 5 and 7?	9 plus 8 = ?	7 and 8 = ?
What is the sum of 12 and 9?	8 plus 6 = ?	11 and 9 = ?
What is the sum of 7 and 4?	10 plus 11 = ?	5 and 12 = ?

2.

Read the following, supplying the required number in place of the sign ( ). Thus, \$9 + \$5 = \$14.

1. 9 dollars + ( ) dollars = 14 dollars.
2. 8 dollars + ( ) dollars = 15 dollars.
3. 7 dollars + ( ) dollars = 19 dollars.
4. 5 cents + ( ) cents = 16 cents.
5. 3 cans + ( ) cans = 8 cans.
6. 9 muffs + ( ) muffs = 20 muffs.
7. 6 cards + ( ) cards = 17 cards.
8. 10 tons + ( ) tons = 15 tons.
9. 8 books + ( ) books = 13 books.
10. 5 men + ( ) men = 14 men.
11. 7 leaves + ( ) leaves = 19 leaves.
12. 11 bottles + ( ) bottles = 23 bottles.
13. 7 + ( ) = 13.
14. 12¢ + ( ) ¢ = 20¢.
15. 11 + ( ) = 17.
16. 5 + ( ) = 19.
17. 6 + ( ) = 18.
18. 7 + ( ) = 13.
19. 9 desks + ( ) desks = 18 desks.
20. 10 buds + ( ) buds = 20 buds.
21. \$6 + ( ) \$ = \$15.
22. 7 dolls + ( ) dolls = 11 dolls.
23. 5 houses + ( ) houses = 17 houses.
24. 7 beans + ( ) beans = 14 beans.
25. 8 pictures + ( ) pictures = 21 pictures.
26. 12 horses + ( ) horses = 19 horses.
27. 6 cars + ( ) cars = 24 cars.
28. 9 pens + ( ) pens = 17 pens.
29. 4 trees + ( ) trees = 15 trees.

3.

1. Mr. Williams received 16 business letters one day and 10 the next; how many letters did he receive in the two days?

2. What is the sum of 25 and 22?

NOTE.—Let it be remembered that 25 consists of 2 tens and 5 units or 20 + 5, and 22 consists of 2 tens and 2 units or 20 + 2. 2 tens and 2 tens are 4 tens or 40, and 5 units and 2 units are 7 units. 40 and 7 are 47, the sum.

3. I spent $26 for a dress and $23 for a cloak; how much did I spend for both?

4. Frank is 20 years old and his brother is 24; what is the sum of their ages?

5. Thomas had 32 marbles and his brother gave him 31 more; how many had he then?

6. Mr. Clinton bought oranges for 35 cents and lemons for 30 cents; what did he pay for both?

7. Rebecca paid 38 cents for soap and 31 cents for sugar; what did she pay for both?

8. If a table is 34 inches long and 30 inches wide, how many inches half-way around it?

9. Muriel had 30 cents and Enid had 25; how many cents had both?

10. I deposited $26 in a bank and that was $32 less than my neighbor put in; how many dollars did my neighbor put in?

11. Carrie had 24 carnations and Bertha had 35; how many had both?

12. Robert paid $25 for doves and $23 for chickens; what did the doves and chickens cost him?

13. If a man is now 35 years old, how old will he be in 30 years?

4.

1. Helen had 11 plants in one room, 10 in another and 12 in another; how many plants had she in the three rooms?

SOLUTION.—She had the sum of 11 plants, 10 plants and 12 plants. 11 plants and 10 plants are 21 plants and 12 plants are 33 plants. Therefore, she had 33 plants in the three rooms.

2. In a store there are 12 men, 4 boys and 7 girls at work; how many persons working in the store?

3. A man gave $8 for a coat, $3 for shoes and $9 for board; how much did he give for all?

4. In November there are 30 days and in December 31 days; how many days in both months?

5. A lady spent $20 for a shawl, $14 for a muff, and had $11 left; how many dollars had she at first?

6. Mr. Baker paid $60 for a buggy, $11 for repairing it and $5 for painting it; how much did it cost him?

7. Bought a horse for $70; for how much must I sell him to gain $29?

8. I have in my purse a quarter, a five-cent piece and 2 cents; how much have I?

9. How far around a room that is 12 feet square?

10. At an entertainment 29 pupils gave an exhibition of the Del Sartean system of exercise, 9 recited poetry and 8 sung; how many took part in the entertainment?

11. A pole stands 11 feet in the mud, 15 feet in the water and 21 feet above the water; what is its length?

12. I paid 30 cents for oranges, 35 cents for coffee and 75 cents for flour; how much money did I expend?

SOLUTION.—I paid the sum of 30 cents, 35 cents and 75 cents. 30 cents and 35 cents are 65 cents and 75 cents are 140 cents or $1.40. Therefore, I expended $1.40.

NOTE.—In expressing dollars and cents by figures, a period, called the *decimal point*, is placed between the dollars and cents.

Thus, 152¢ equals $1.52, and is read One dollar and fifty-two cents.

When the number of cents is less than ten, a cipher must be written before the number of cents.

Thus, 4 cents, written $.04.

13. Jane earned 24 cents, Mary 25 cents and Minnie 32 cents; how much money did all earn?

14. George spent $.20 one day, $.23 the second day and $.27 the third day; how much money did he spend in the three days?

15. I spent 21 cents for soap, 35 cents for gasoline and 40 cents for sugar; how much did I spend for all?

16. Read the following: $2.10; $4.75; $3.76; $5.89; $10.25.

17. Express the following: Four dollars and seventy-five cents.

18. Two dollars and three cents; seventeen dollars and nine cents.

5.

1. How many are 2 and 2? 22 and 2? 32 and 2? 62 and 2? 92 and 2?

2. How many are 3 and 3? 13 and 3? 43 and 3? 53 and 3? 83 and 3?

3. How many are 4 and 4? 34 and 4? 54 and 4? 64 and 4? 74 and 4?

4. How many are 5 and 5? 25 and 5? 35 and 5? 45 and 5? 55 and 5?

5. How many are 6 and 6; 16 and 6? 26 and 6? 86 and 6? 96 and 6?

6. How many are 7 and 7? 27 and 7? 37 and 7? 57 and 7? 77 and 7?

7. How many are 8 and 8? 28 and 8? 38 and 8? 48 and 8? 68 and 8?

8. How many are 9 and 9? 39 and 9? 49 and 9? 59 and 9? 69 and 9?

9. How many are 10 and 10? 20 and 10? 50 and 10? 60 and 10? 90 and 10?

10. How many are 1 and 7? 11 and 7? 21 and 7? 31 and 7?

11. How many are 2 and 7? 42 and 7? 52 and 7? 62 and 7?

12. How many are 3 and 7? 23 and 7? 73 and 7? 83 and 7?

13. How many are 4 and 7? 14 and 7? 34 and 7? 64 and 7?

14. How many are 5 and 7? 25 and 7? 45 and 7? 75 and 7?

15. How many are 6 and 7? 36 and 7? 66 and 7? 86 and 7?

16. How many are 7 and 17? 47 and 7? 57 and 7? 77 and 7?

17. How many are 8 and 7? 58 and 7? 38 and 7? 78 and 7?

18. How many are 9 and 7? 69 and 7? 29 and 7? 59 and 7?

19. How many are 10 and 7? 30 and 7? 40 and 7? 60 and 7?

20. How many are 8, 9 and 7? 7, 6 and 8? 6, 7, 8 and 9?

21. What three numbers added make 20? 16? 14?

22. What two numbers added make 47? 57? 67?

6.

As you read the following numbers, mention the given number first. Thus, 4 and 6 are 10, etc.

1.	4	8	( )	13	( )	( )	8	9	( )	( )
	( )	( )	17	( )	14	( )	13	( )	4	( )
	10	15	26	20	26	17	( )	21	11	12

Begin at the right in each number and read the sum. Thus, in (1) say 10, 8. 80 the sum.

	(1)	(2)	(3)	(4)	(5)	(6)	(8)
2.	46	36	687	732	8436	93765	56789
	34	17	572	456	4769	64738	37068
	(8)	(9)	(10)	(11)	(12)	(13)	(14)
	13	19	23	659	987	3064	123456
	12	24	37	478	789	9478	987654

Add the following by columns, using combinations:

	(15)	(16)		(17)	(18)	(19)
3.	26	437	12	5426	98765	123456
	39	865		9584	56789	987654
	84	978	15	3859	19345	678932
	65	347		2468	54321	456789
	46	567	10	1357	86457	789654
	85	483		7985	79862	976543

	(20)	(21)	(22)
4.	$ 876.54	$5276.76	$1200.50
	14.75	4382.98	987.62
	3000.25	5386.42	364.25
	398.62	4268.48	387.28

NOTE.—Teachers should write similar exercises on the blackboard occasionally to test the ability of the class to add *rapidly* and *accurately*.

After adding the above numbers (15) to (19) inclusive, in the usual way, vertically, add the numbers in each example in a horizontal direction. Thus, example (17), last number 5, 8, 9, 7 say 5, 13, 22, 29.

SUBTRACTION.

Introductory Exercises.

1. If I have 4 books and sell 2 of them, how many have I left?

Answer. I have left the difference between 4 books and 2 books which is 2 books.

2. If a boy had 5 cents and spent 3 of them, how many had he left?

3. 3 from 4 leaves how many?

Ans. 3 from 4 leaves 1.

4. 2 from 5 leaves how many? 3 from 5?

5. Can you take 3 books from 4 pencils? Why not?

6. Can you take 3 books from 5 books? Why?

7. What are like numbers?

Definitions.

1. Subtraction is the process of finding the difference between two numbers of the same kind.

2. Difference or Remainder is the result found by subtracting.

3. The Minuend is the number from which we subtract.

4. The Subtrahend is the number to be subtracted.

5. The Sign of Subtraction is a short, horizontal line, —, and is read *minus*. When it is placed between two numbers it shows that the one after it is to be taken from the one before it.

Thus, 6 — 3 is read 6 minus 3, and means that 3, the subtrahend, is to be subtracted from 6, the minuend.

In the expression, $6 - 2 = 4$, 6 is the *minuend*, 2 the *subtrahend*, and 4 the *difference* or *remainder*.

NOTE.—To find the *Minuend*, add Remainder and Subtrahend. To find the *Subtrahend*, subtract Remainder from Minuend.

1.

PROBLEMS.

1. Mary's uncle gave her 5¢; if she spent 2¢, how many had she left?

SOLUTION.—She had left the difference between 5¢ and 2¢, which is 3¢. Therefore, she had 3 cents left.

2. A farmer sold his pig for $7 and his sheep for $2; how much more did he receive for the pig than for the sheep?

3. Harry earned 8 cents and spent 2 of them; how many had he left?

4. If a boy who had 10 papers lost 2 of them, how many had he left?

5. In a group of 7 wigwams, 3 were blown down by a storm; how many remained standing?

6. A little boy agreed to make 8 wooden bird-cages; if he has finished three of them, how many more has he to make?

7. There were 9 girls at a party; if all but 3 have gone home, how many have gone home?

8. Jeremiah had 13 chickens and sold 3 of them; how many has he left?

9. Martha gave away 4 of her 9 bananas; how many has she left?

10. Miss Tripp bought 8 blotters and gave away 4 of them; how many has she left?

11. It rained 3 days out of 10; how many days were pleasant?

12. If I sold all but 4 of 11 rabbits; how many did I sell?

13. If two boys caught 8 fish and one of them caught 5, how many did the other catch?

14. Out of 15 words, James missed 5; how many words did he spell correctly?

15. Winnie paid Mr. Pringle 5 cents for buttons; how much change ought she to receive from 10 cents?

16. Mr. Stout bought 11 books and Mr. Slim bought 5; how many more did Mr. Stout buy than Mr. Slim?

17. Helen's father gave her 13 cents to spend; if she spent 5 cents, how many had she left?

PRACTICE EXERCISES.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
18.	From	6	7	10	7	8	15	8	10	9	10	7	9
	Take	2	3	5	4	3	5	4	4	6	3	2	2
19.	From	6	7	8	9	10	11	12	15	13	7	8	5
	Take	3	5	4	2	3	5	4	5	4	3	2	3

20. Morse has 11 tops and Fred has 6; how many more has Morse than Fred?

21. Sarah has 9 rooms to clean; how many has she to finish when she has cleaned 6?

22. Irene is now 10 years of age; how old was she 6 years ago?

23. If Arthur is 12 years of age and Ferdinand 6, what is the difference between their ages?

24. If a man who earned $15 a week spent $6 a week, how much did he save in that time?

25. Irving bought a clock and a lamp for $16; if the lamp cost him $6, what was the cost of the clock?

26. Jane bought a couch for $6 and sold it for $10. What was her gain?

27. There are 13 birds in the barn; if 7 of them flew away, how many remained in the barn?

28. A cat caught 7 mice one day; how many more must she catch to make the number 12?

29. Thomas has saved $8; how much does he lack of $12?

30. James bought a writing book for 8 cents; how much change ought he to receive from 10 cents?

31. Bought a piece of cloth for $8 and sold it for $13; how many dollars did I gain?

32. Jane picked up 8 quarts of chestnuts; how many more quarts will make the number of quarts 15?

33. What must be added to 2, 3, 8, 4, 6, 5, 7, 9 to make 13?

34. What must be added to 3, 5, 7, 6, 4, 2, 8, 9 to make 16?

35. What must be added to 3, 7, 8, 6, 1, 5, 4, 2, 9 to make 14?

36. Count from 50 back to 2 by 2's.

37. Count from 40 back to 0 by 4's.

38. Count from 36 back to 0 by 3's.

39. Count from 45 back to 0 by 5's.

40. From 16 take 8; from 18 take 9; from 14 take 7.

41. Take 5 from 13; 6 from 12; 7 from 15; 8 from 13.

Practice Exercises.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
42.	From	16	7	5	14	13	12	15	12	10	9	11	8
	Take	8	2	3	7	6	5	5	7	8	3	8	2

43.

From	8	10	7	12	24	13	10	12	10	15	15	10
Take	3	4	2	5	7	6	1	8	0	8	6	7

44. Mr. Frank had 21 bushels of potatoes and sold 10 bushels; how many bushels had he left?

45. Frank paid 17 cents for a top and sold it for 10 cents; how much did he lose?

46. Josiah is 23 years of age and his brother is 10 years younger; what is his brother's age?

47. If Walter is 22 years old, what was his age 11 years ago?

48. How much change should Ida receive from $20 if her dress cost her $11?

49. If the minuend is 18 and the subtrahend is 11, what is the remainder?

50. If the minuend is 15 and the remainder 11, what is the subtrahend?

51. If the sum of two numbers is 18 and one of the numbers is 12, what is the other number?

2.

Addition and Subtraction.

1. From a cane 23 inches long, 9 inches were cut at one time and 4 at another; how many inches were left?

Solution.—As many inches were left as the difference between 23 inches and the sum of 9 inches and 4 inches. 9 inches and 4 inches are 13 inches, and 23 inches minus 13 inches equals 10 inches. Therefore, there were 10 inches left.

2. A man having 20 tubs of butter sold 3 tubs at one time and 8 at another; how many tubs had he left?

3. From a pole 24 feet long, 7 feet were cut at one time and 12 at another; how many feet remained?

4. Martha bought 8 cents worth of candy, 5 cents worth of ribbon and two one-cent stamps; how much had she left of 20 cents?

5. Augusta was 5 years of age 4 years ago; what is the difference between her age at present and that of Muriel who is 18 years of age?

6. Mr. Generous gave his son 10 dollars for a birthday present. The son bought skates for 3 dollars, gloves for 3 dollars, and a tie for 1 dollar. How much had he left?

7. Irene has 21 examples to work. She finished 7 the first hour and 5 the second hour; how many more has she to work?

8. Henry spent $5 for board and $3 for a room; John spent $6 for board and $4 for a room; how much more did John spend than Henry?

SOLUTION.—As much more as the difference between the sum of $5 and $3, and the sum of $6 and $4. The sum of $5 and $3 is $8, and the sum of $6 and $4 is $10. The difference between $10 and $8 is $2. Therefore, John spent $2 more than Henry.

9. I rode 6 miles on Monday and 10 miles on Tuesday. Wednesday I rode 4 miles on my return and Thursday 8 miles on my return. How far from the starting place was I at the end of Thursday?

10. George earned $9 and David earned $12; the two brothers bought a Christmas present costing $6 for their mother and one costing $4 for their sister; how much money had they left?

11. I paid $5 for a pair of shoes, $3 for a book and $4 for a hat; my friend spent $7 for a hat and $15 for a cape; how much more did she spend than I?

12. I bought 27 boxes of velvet candy; how many boxes had I left after giving Esther 9 boxes and Ruth 6 boxes?

3.

Read each of the following and give results.

1. To 6 add 4, add 3, subtract 4, add 8, subtract 4.
2. 8 plus 5, plus 4, minus 6, minus 3, plus 5.
3. From 15 subtract 8, add 7, add 5, subtract 11.

4. $9 + 8 + 1 - (\ ) = 8$.	*13.* $7 - 7 + (\ ) = 8$.
5. $6 + 9 - 5 = (\ ) + 4$.	*14.* \$9 — \$3 — \$2 = ( ) + \$1.
6. $8 - 4 + 12 = (\ ) + 7$.	*15.* $24 - 12 + (\ ) = 15$.
7. $9 + 3 - 8 + 5 = ?$	*16.* $10 + 11 - 12 - 4 = ?$
8. $6 + 6 - 5 - 3 = ?$	*17.* $9 - 8 + 7 - 3 = ?$
9. $10 - 6 + 4 + 2 = ?$	*18.* $10 - 5 + 3 + 8 = ?$
10. $6 + 3 - 9 + 5 = ?$	*19.* $4 + 5 - 6 - 3 = ?$
11. $8 + 2 - 6 - 3 = ?$	*20.* $8 + 7 - 9 - 4 = ?$
12. $12 + 3 - 9 - 6 = ?$	*21.* $15 - 10 + 11 + 2 = ?$

Practice Exercises.

Find the results of the following:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
22. From	12	17	18	12	19	20	16	18	19	16
Take	7	8	10	4	12	10	9	11	7	9
And add	4	5	9	13	6	11	4	6	7	10

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
23. From	19	20	10	15	14	13	17	18	19	15
Take	12	6	7	11	9	7	9	8	7	6
And add	11	12	3	4	5	6	4	5	6	7

4.

1. Add 34 and 27.

This example and similar ones may be added as follows: $34 + 20 + 7 = 61$.

2. Add 27, 36 and 24.

$27+30+6+20+4$.

3. Add 42 and 65.
4. Add 39 and 24.
5. Add 16 and 84.
6. Add 22, 34 and 15.
7. Add 25, 42 and 33.
8. Add 32, 36 and 13.
9. Add 41, 33 and 24.
10. Add 74 and 25.
11. Add 82 and 68.
12. Add 56 and 37.
13. Add 28, 38 and 48.
14. Add 25, 36 and 47.
15. Add 35, 62 and 31.
16. Add 27, 18 and 44.

17. From 71 take 36.

NOTE.—The following method may be used: $71-30-6=35$.

18. From 62 take 35.
19. From 51 take 29.
20. From 43 take 28.
21. From 60 take 37.
22. Take 35 from 53.
23. Take 88 from 93.
24. Take 26 from 42.
25. Take 16 from 31.

26. Find the difference between 62 and 17.
27. Find the difference between 50 and 23.
28. Find the difference between 71 and 49.
29. Find the difference between 56 and 28.
30. Find the difference between 34 and 19.
31. Find the difference between 43 and 27.

5.

1. Julia made 14 Christmas gifts; she gave to her brothers 5, to her sisters 4, and to her mother the remainder; how many did she give to her mother?

2. A pole stands 3 feet in the mud, 4 feet in the water and 7 feet above the water; what is its entire length?

3. Charles Augustus had 70 roses; he gave 20 to Angelina, 30 to Pauline, and kept the rest; how many did he keep?

4. William owned 27 rabbits; he gave 6 to one friend, 9 to another and 12 to another; how many had he left?

5. A man owed one friend 19 dollars, another friend 10 dollars and a third friend 6 dollars; he paid one 10 dollars, another 5 dollars and the third 3 dollars; how much does he still owe?

6. A man, counting his money, found he had 3 ten-dollar bills, 2 five-dollar bills, a two-dollar bill and a one-dollar bill; after paying his debts he had left 3 two-dollar bills and 1 five-dollar bill; what were his debts?

7. Eliza bought a brush for 10¢, a spool of thread for 4¢, a spool of silk for 10¢, a paper of agate buttons for 5¢; how much change should she receive from 50¢?

8. George had a ten-cent piece, a five-cent piece and 2 cents. He bought a kite for 4 cents and a toy pistol for 7 cents; how many cents had he left?

9. A boy had 15 apples. He ate 4 and sold 5; how many had he left?

10. Ruth is 7 years of age and Muriel is 5 years older; in how many years will Muriel be 18 years of age?

11. Two men started from the same point and traveled in opposite directions, one at the rate of 29 miles an hour, the other at the rate of 26 miles an hour; how far apart will they be at the end of an hour?

12. In a certain orchard there are 9 cherry trees, 11 peach trees and 29 plum trees and as many apple trees as there are cherry, peach and plum trees; how many apple trees are there?

13. How many trees in the orchard mentioned in example number 12?

14. John sold apples for 50 cents, corn for 20 cents and eggs for 30 cents; he received in exchange one drum

of figs worth 60 cents and the rest in cash; how much did he receive in cash?

15. Mr. Westbury is 78 years old; how old was he 19 years ago? How old will he be in 15 years?

16. Mr. Stiles bought a book-case for 25 dollars, a chair for 13 dollars, a lamp for 12 dollars and a picture for 6 dollars; how much less than 60 dollars did he spend?

17. Lucy is 15 years old, Minnie is 14 and Elizabeth is 19; what is the sum of their ages?

18. I spent 27 cents one day, 37 another and 17 another; how much more than 75 cents did I spend?

19. Four men bought a bicycle for $80; the first paid $35, the second $20, and the third $15; how much did the fourth pay?

20. A man traveled 24 miles one day, and 26 miles the second day, and on the third day he traveled back 15 miles; how many miles was he from the place from which he started?

21. On a certain bush there were 42 roses. 18 of them were picked and 14 were blown off; how many roses were left on the bush?

22. A man died at the age of 70 years, having been married 35 years; what was his age when he married?

23. A man bought a watch for $35, a chain for $12, and a pin for $5, and he sold them all for $60; did he gain or lose by the transaction, and how much?

6.

1. 14 less 8 are how many? 19 less 11 are how many?

2. 65 less 26 are how many? 98 less 79 are how many?

3. 25 less 12 plus 17 are how many?

4. 19 plus 7 less 14 are how many?

5. 35 less 18 less 17 are how many?

6. 16 plus 7 less 7 less 4 are how many?

7. 7 plus 5 less 4 plus 5 less 3 are how many?

8. 23 plus 16 less 19 plus 3 less 5 are how many?

9. 14 plus 16 plus 5 less 15 plus 3 are how many?

10. 17 less 8 plus 9 less 3 plus 8 are how many?

11. 16 less 12 plus 22 less 6 less 3 are how many?

12. 42 less 18 less 10 plus 5 plus 4 are how many?

13. From the sum of 24 and 36 take the difference between 28 and 19.

14. From the difference between 99 and 77 take the difference between 88 and 69.

15. To the sum of 23 and 14 add the difference between 18 and 9.

16. From the difference between 33 and 14 take the sum of 15 and 4.

Give results only.

Practice Exercises.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
17.	From	18	17	25	21	36	15	42	37	21	42	28	52
	Take	7	9	19	17	29	7	23	18	9	28	19	28
18.	Add	16	17	19	21	42	19	27	49	73	36	56	43
		17	18	17	36	29	29	37	23	89	48	87	97

Read and supply the required term.

19.—	*20.*—	*21.*—
23 + 10 = ?	60 + ( ) = 80.	25 — ( ) = 15.
48 — 20 = ?	( ) + 20 = 50.	45 — 20 = ?
80 + ( ) = 100.	20 + ( ) = 60.	( ) — 10 = 40.
50 — 20 = ?	80 — ( ) = 50.	( ) + 30 = 50.

MULTIPLICATION.

Introductory Exercises.

1. Henry, George and John have 2 books apiece; how many books have they all?

Answer. They have the sum of 2 books taken 3 times, or 3 times 2 books, which are 6 books.

2. I bought 2 pencils for 3 cents each; how much did they cost me?

3. If a man earns $3 a day, how much can he earn in 3 days?

4. How many are three 2's? How many are 3 times 2?

5. A man walked 4 miles an hour for 3 hours; how far did he walk? Three 4's are how many?

6. A man walked 4 miles an hour for 3 hours; how far did he walk? Three 4's are how many?

7. What is the difference between three 4's and four 3's?

8. Add by 2's to 20. By 3's to 30. By 4's to 40.

9. Multiply each of the following numbers by 2: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

10. What sum is produced by taking $4 five times?

11. What is produced by taking $5 as many times as there are units in 3?

Definitions.

1. Multiplication is the process of repeating one number as many times as there are units in the other. It is a short method of finding the sum of equal numbers.

2. The Multiplicand is the number to be repeated.

3. The Multiplier is the number which shows how many times the multiplicand is to be repeated.

4. The PRODUCT is the result obtained by multiplying.

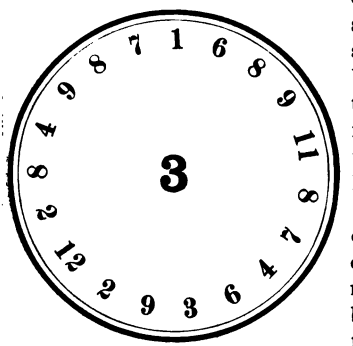
5. The FACTORS are the Multiplicand and Multiplier. They are called factors because they produce the product. Thus, 3 and 2 are *factors* of 6. 2 and 5 are factors of 10.

6. The SIGN of MULTIPLICATION is an oblique cross, $\times$. It is read *multiplied by* or *times*. When it is placed between two numbers it shows that they are to be multiplied together. Thus, 3×2 is read 3 multiplied by 2 or 2 times 3.

3 multiplied by 2 equals 6. In this expression 3 is the *multiplicand*, 2 the *multiplier* and 6 the *product*.

The expression, 3×2, may be read 2 times 3, or 3 times 2, since changing the order of the factors does not change the product.

PRACTICE DIAGRAM.



Make a circle similar to the one used in Addition. Place a number in the center, as 3; and, commencing at any number, as at 4, and going around the circle to the right or left, require the pupil to repeat the table to correspond with the lesson of the day.

Change the number in the center and require the tables of the preceding lessons to be reviewed and the answers to be given at sight when pointed to by the teacher.

To give variety, use the figures in the margin for multipliers and the central figure for the multiplicand.

A few minutes of each day's recitation, spent in exercises of this kind, will soon make the pupil familiar with all the tables. *Let this work be thorough.*

Multiplication Table.

1 × 0 = 0	2 × 0 = 0	3 × 0 = 0	4 × 0 = 0
1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 0 = 0	6 × 0 = 0	7 × 0 = 0	8 × 0 = 0
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 0 = 0	10 × 0 = 0	11 × 0 = 0	12 × 0 = 0
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

1.

Problems.

1. What will 2 tons of hay cost at $9 a ton?

Solution.—Since one ton cost $9, two tons will cost 2 times $9, or $18. Therefore, 2 tons of hay at $9 a ton will cost $18.

2. How many pints in 7 quarts, if in one quart there are 2 pints?

3. How many mills in 3 cents, if in one cent there are 10 mills?

4. At 6 cents a pound, what will 3 pounds of sugar cost?

5. If a stage can travel 7 miles an hour, how far can it travel in 3 hours?

6. If 10 yards of cloth make a dress, how many yards will it take to make 3 dresses?

7. James is 4 years old and his brother George is 3 times as old; how old is George?

8. In an orchard there are 3 rows of plum trees with 6 trees in a row; how many plum trees in the orchard?

9. What will 3 pineapples cost at 11 cents each?

10. Florence worked 5 examples and Mary worked 3 times as many; how many did Mary work?

11. Multiply each of the following by 2: 8, 9, 6, 5, 4, 3, 7, 1, 10, 12, 11.

12. Multiply each of the following by 3: 12, 11, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1, 0.

13. Herbert is 7 years old and his father is 4 times as old; how old is his father?

14. If Seth earned $9 a week, how much did he earn in 4 weeks?

15. What will 4 coats cost at $5 each?

16. How many weeks in 9 months, if in one month there are 4 weeks?

17. If one piece of ice cost me 10 cents, what will 4 pieces of the same size cost?

18. How many pecks in 12 bushels, if in one bushel there are 4 pecks?

19. Multiply each of the following by 4: 11, 3, 2, 1, 0, 8, 9, 7, 6, 4, 12.

20. Multiply each of the following by 5: 11, 10, 3, 5, 8, 9, 7, 6, 0, 2, 4, 12.

21. Multiply each of the following by 6: 1, 3, 5, 7, 9, 8, 2, 12, 4, 6, 10, 0, 11.

22. What will 5 bags of nuts cost at 8 cents a bag?

23. At $3 a pair, what will 5 pairs of skates cost?

24. What will 5 car tickets cost at 5 cents each?

25. Lucy rode 6 miles on Monday and 5 times as far on Wednesday; how far did she ride on Wednesday?

26. If one boy can eat 4 pieces of bread and butter, how many pieces can 5 boys eat in the same time?

27. Mr. Lee took 7 pictures and Mr. Dumble 5 times as many; how many did Mr. Dumble take?

28. What will 7 yards of ribbon cost at 12 cents a yard?

29. What will 7 spools of silk cost at 8 cents a spool?

Practice Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
30. Multiply	8	3	4	9	10	5	7	5	6	7	8	9
by	5	5	6	6	5	6	5	5	6	6	6	5
31. Multiply	3	3	7	2	8	8	9	7	5	12	4	11
by	6	7	7	6	6	7	7	6	7	6	7	7

32. How far can a horse travel in 8 days, if he can travel 7 miles a day?

33. What will 7 boxes cost at 6 cents each?

34. What will 7 barrels of flour cost at $8 a barrel?

35. Mr. Cole saved $4 a week and earned 7 times as much; how much did he earn?

36. At $5 each, what will 7 books cost me?

37. If one dish of ice cream cost 10 cents, what will 7 dishes cost?

38. Multiply each of the following by 7: 1, 3, 5, 7, 9, 11, 2, 0, 4, 6, 8, 10, 12.

39. Multiply each of the following by 8: 0, 1, 2, 4, 5, 6, 12, 11, 10, 9, 8, 7.

40. If 7 men can do a piece of work in 6 days, how long will it take one man to do it?

SOLUTION.— Since 7 men can do it in 6 days, it will take one man 7 times 6 days, or 42 days to do it. Therefore, it will take one man 42 days to do the work.

41. What will 7 yards of factory cost at 11 cents a yard?

42. If one writing-pad cost 9 cents, what will 8 such pads cost?

43. If 8 girls sold 8 tickets each, how many did all sell?

44. What will 8 pounds of Cottolene cost at 9 cents a pound?

45. At 12 cents a dozen, how much ought I to pay for 8 dozen eggs?

46. How many quarts are there in 8 pecks, if in one peck there are 8 quarts?

Practice Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
47. Multiply	3	7	5	6	4	3	6	8	5	4	7	9
by	7	8	7	8	7	8	7	7	8	8	7	8
48. Multiply	9	0	6	10	12	4	2	11	7	5	0	8
by	9	8	8	9	9	8	9	8	8	8	9	9

49. If in one bale there are 5 bundles, how many bundles are there in 9 bales?

50. What will 9 yards of wire cost at 4 cents a yard?

51. How many days are there in 9 weeks, if in one week there are 7 days?

52. If Mary can earn 11 cents an hour, how much can she earn in 9 hours?

53. What will 9 yards of ribbon cost at 12 cents a yard?

54. If a boy can work 11 examples a day, how many can he work in 6 days?

55. Erwin is 2 years of age; if Philip is 11 times as old, how old is Philip?

56. How many trees are there in 11 rows, if in one row there are 12 trees?

57. What will 12 loaves of bread cost at 4 cents a loaf?

58. Martha sold 6 cents worth of paper flowers and Grace sold 12 times as many; how many cents worth did Grace sell?

59. Mary paid 5 cents for a pencil and 12 times as much for a book; what did she pay for a book?

60. Multiply each of the following by 9: 3, 4, 0, 6, 7, 10, 9, 5, 8, 2, 12, 11.

61. Multiply each of the following by 10: 1, 5, 9, 8, 6, 12, 4, 0, 3, 7, 10, 11.

Practice Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
62. Multiply	5	6	7	8	9	10	11	12	2	9	10	6
by	3	5	4	7	9	9	6	4	3	2	0	7
63. Multiply	7	8	9	10	11	12	8	8	7	11	10	5
by	2	3	4	5	6	7	8	9	9	12	11	8

64. What will 10 yards of ribbon cost at 12 cents a yard?

65. If 12 men can do a piece of work in 6 days, how long will it take one man to do it?

66. James earned $3 a week and his father earned 12 times as much; how much did his father earn?

67. What will 5 capes cost at $12 each?

68. How long will it take one man to accomplish a piece of work that 12 men can do in 12 days?

69. What will be the cost of 12 yards of factory, if one yard cost 9 cents?

70. If you work 8 examples an hour, how many can you work in 12 hours?

71. What will 12 clocks cost at $7 apiece?

72. Multiply each of the following by 11: 8, 2, 5, 6, 9, 7, 3, 15, 4, 0, 10, 12, 11.

73. Multiply each of the following by 12: 1, 12, 2, 11, 3, 10, 4, 9, 5, 8, 6, 7, 0.

2.

1. There are 6 boats on the lake and 4 boys in each boat; how many boys on the lake?

2. How many feet in 9 yards, if in one yard there are 3 feet?

3. How many school days in 5 weeks?

4. Mrs. Brown gave 6 dollars for her daughter's dress and 6 times as much for her own; what did Mrs. Brown's dress cost?

5. I paid 3 dollars for a hat and 7 times as much for a suit of clothes; what did hat and clothes cost me?

SOLUTION.—They both cost me the sum of three dollars and the product of 7 times 3 dollars. 7 times 3 dollars are 21 dollars; and 3 dollars and 21 dollars are 24 dollars. Therefore, I gave 24 dollars for hat and clothes.

6. If Janet deposits $4 in the bank each month, how much will she have in the bank at the end of 1 year or 12 months?

7. How many cents in 10 dimes?

8. In a school-room there are 6 rows of seats with 5 seats in a row; how many seats in the room?

9. How many feet have 5 flies, if each fly has 6 feet?

10. If one man can build 2 rods of fence in one day, how many rods can 5 men build in the same time?

3.

1. If 3 men can do a piece of work in 1 day, how many days will it take one man to do it?

SOLUTION.—Since 3 men can do it in one day, it will take one man 3 times one day to do it. 3 times one day are 3 days. Therefore, it will take 1 man 3 days to do it?

2. If 4 men can do a piece of work in 5 days, how long will it take one man to do it?

3. If 6 men can do a piece of work in 7 days, how many men will it take to do it in one day?

4. If a quantity of provisions will serve 3 men 5 days, how many men will it serve one day?

5. A man had a piece of work to perform which 7 men could do in 5 days, but it was necessary that the whole should be completed in one day; how many men must he employ?

6. If the interest of one dollar is 6 cents a year, what will be the interest of 10 dollars for the same time?

7. If a man can earn 8 shillings a day, how many shillings can he earn in 11 days?

8. Thomas saved 6 dollars a week for 12 weeks; how much money had he then saved?

9. What will 9 brush-brooms cost if one cost 12 cents?

10. If one dozen eggs cost 14 cents, what will 7 dozen cost?

4.

1. A boy bought 4 pints of chestnuts at 3 cents a pint, and sold them for 20 cents; how much did he gain?

SOLUTION.—He gained the difference between 20 cents and the product of 4 times 3 cents. 4 times 3 cents are 12 cents, and 20 cents less 12 cents are 8 cents. Therefore, he gained 8 cents.

2. A woman had a jar of butter containing 24 pounds; she used 6 pounds one week, 8 pounds the second week and the remainder the third week; how much did she use the third week?

3. Two men travel in the same direction, one at the rate of 3 miles an hour and the other at the rate of 5 miles an hour; how far apart are they at the end of 4 hours?

4. A farmer bought 12 pounds of sugar at 5 cents a pound, and gave in payment 3 dozen eggs at 11 cents a dozen and the rest in cash; what was his cash payment?

5. How many horns have 4 yoke of oxen?

6. Two men travel in opposite directions, one at the rate of 5 miles an hour and the other at the rate of 7 miles an hour; how far apart will they be at the end of 6 hours?

7. William bought 5 bars of soap at 8 cents a bar and 6 pounds of sugar at 5 cents a pound; how much did he pay for soap and sugar?

8. Ned earned $7 a week for 6 weeks and spent $3 a week for the same time; how much did he save in 6 weeks?

9. What is the difference in cost between 8 chairs at $8 each and 7 chairs at $9 each?

10. Raymond has 3 marbles, Harry has 4 more than Raymond, and Frank has 4 times as many as Raymond and Harry together; how many has Frank?

SOLUTION.—Frank has as many marbles as 4 times the sum of 3 marbles and 3 plus 4 marbles. 3 marbles and 4 marbles equal 7 marbles, and the sum of 3 marbles and 7 marbles is 10 marbles. 4 times 10 marbles are 40 marbles. Therefore, Frank has 40 marbles.

11. John has 9 dollars and Joseph has 6 times as many lacking 4 dollars; how many dollars has Joseph; how many dollars have both?

12. In one of two school-rooms there are 7 rows of seats with 7 seats in a row; in the other there are 5 rows with 9 seats in a row; which of the two rooms holds the greater number of pupils and how many more?

13. Clyde earned $8 a week for 6 weeks and Ralph $10 a week for 5 weeks; what is the difference between the sums earned by each?

14. Mary bought 9 dozen napkins at $3 a dozen and Nellie bought 5 dozen at $5 a dozen; find the difference between the sums expended by each?

15. Levi is 8 years old and Solomon is 4 years older; how old is their father, who is twice as old as the sum of their ages?

16. Grace is 12 years old and Reuben is 4 years younger; what is 12 times the difference between their ages?

17. Nellie is 10 years old and Walter is 3 years younger; how old is Robert, who lacks 2 years of being as old as Nellie and Walter together?

5.

Promiscuous Multiplication Table.

1.	2 times 8 are how many?	*13.*	$5 \times 6 = ?$
2.	3 times 9 are how many?	*14.*	$9 \times 9 = ?$
3.	4 times 8 are how many?	*15.*	$7 \times 4 = ?$
4.	7 times 5 are how many?	*16.*	$3 \times 9 = ?$
5.	9 times 4 are how many?	*17.*	$5 \times 7 = ?$
6.	6 times 3 are how many?	*18.*	$8 \times 5 = ?$
7.	4 times 9 are how many?	*19.*	$10 \times 10 = ?$
8.	6 times 9 are how many?	*20.*	$9 \times 5 = ?$
9.	5 times 8 are how many?	*21.*	$6 \times 4 = ?$
10.	7 times 6 are how many?	*22.*	$8 \times 3 = ?$
11.	8 times 8 are how many?	*23.*	$11 \times 11 = ?$
12.	9 times 7 are how many?	*24.*	$12 \times 8 = ?$

25. 3 times 11 dollars, plus 7 dollars, are how many dollars?

26. 9 times 7 dollars, minus 13 dollars, are how many dollars?

27. 5 times 12 pecks, plus 15 pecks, are how many pecks?

28. 6 times 8 quarts, plus 6 quarts, are how many quarts?

29. 7 times 4 pints, minus 12 pints, are how many pints?

30. 7 times 7 cents, plus 11 cents, are how many cents?

31. 9 times 4 dimes, minus 16 dimes, are how many dimes?

32. 9 times 8 bushels, minus 9 bushels, are how many bushels?

33. 8 times 7 yards, plus 14 yards, are how many yards?

34. 5 times 5 feet, plus 15 feet, are how many feet?

35. 3 times 8 rods, minus 6 rods, are how many rods?

36. 8 times 8 dollars, minus 34 dollars, are how many dollars?

As you read, supply the required term in each of the following:

37. 8 times ( ) = 56.
38. 7 times ( ) = 49.
39. 4 times ( ) = 36.
40. 6 times ( ) = 42.
41. 5 times ( ) = 45.
42. 9 times ( ) = 27.
43. 2 times ( ) = 24.
44. ( ) times 4 = 32.
45. ( ) times 9 = 18.
46. 7 times ( ) = 42.
47. 8 times ( ) = 64.
48. ( ) times 6 = 72.
49. 9 times ( ) = 81.
50. ( ) times 11 = 121.
51. 6 times ( ) = 36.
52. 7 times ( ) = 21.
53. 12 times ( ) = 144.
54. 9 times ( ) = 63.
55. 11 times ( ) = 121.
56. 8 times ( ) = 24.
57. 12 times ( ) = 84.
58. 11 times ( ) = 132.
59. 6 times ( ) = 72.
60. 8 times ( ) = 72.
61. 5 times ( ) = 35.
62. 12 times ( ) = 96.
63. 6 times ( ) = 54.
64. 5 times ( ) = 35.
65. 9 times ( ) = 108.
66. 10 times ( ) = 120.
67. 4 times ( ) = 48.
68. 9 times ( ) = 54.
69. 7 times ( ) = 49.
70. 7 times ( ) = 28.
71. 4 times ( ) = 32.
72. 12 times ( ) = 108.

6.

The PARENTHESIS shows that the numbers included within it are to be considered as one number.

Thus, $(6-4)\times 5$ shows that $6-4$ or 2 is to be multiplied by 5.

The VINCULUM may be used instead of the Parenthesis. Thus, $\overline{6-4}\times 5$ may be used instead of $(6-4)\times 5$.

PRACTICE EXERCISES.

1. $(4+5)-6+4=?$
2. $(5\times 9)+5-10=?$
3. $(3\times 2)+9-8=?$
4. $(8\times 9)-12-10=?$
5. $\overline{3\times 4\times 5}+4=?$
6. $7+8-\overline{3\times 5}=?$
7. $9+7-13+5=?$
8. $(6\times 8)-(2\times 9)=?$
9. $(6+11)-(3\times 4)=?$
10. $4+9-8-3=?$
11. $(6\times 7)-12+8=?$
12. $4+(7\times 3)-15=?$
13. $(10\times 2)+(5-1)=?$
14. $8+7+3-10=?$
15. $(6\times 6)-6+5=?$
16. $(5\times 5)-5-10=?$
17. $2+3+4+5-10=?$
18. $(3\times 4)-(8\times 3)=?$
19. $12-(3+3)\times 7=?$
20. $(3\times 6)-\overline{4\times 4}=?$
21. $9\times 8=(\ )\times 12?$
22. $6\times 6=9\times ?$
23. $5\times 5=11+?$
24. $9+4=18-?$
25. $18-6=(\ )\times 4=?$
26. $19-11=(\ )+2=?$
27. $(6\times 7)-(8\times 5)=?$
28. $(5\times 4)\times(3+10)=?$
29. $3\times 4\times 8-3\times 3\times 2=?$
30. $\overline{7\times 7}-\overline{6\times 8}=?$
31. $\overline{3+5+4}-\overline{12\times 0}=?$
32. $(8\times 7)-(9\times 4)+6=?$
33. $(5\times 11)-(9\times 4)=?$
34. $(12\times 6)-\overline{8\times 9}+6=?$
35. $84-\overline{8\times 8}-5=?$
36. $77-\overline{10\times 7}+9=?$
37. $(3+9)-(5\times 2)\times 12=?$
38. $(6\times 11)-(12\times 5)=?$
39. $\overline{8+6}\times 7-3=?$
40. $\overline{14-6}\div(2\times 4)=?$
41. $\overline{13-6}+(16\div 4)=?$
42. $(8+7-9)\times(5\times 4)=?$
43. $\overline{9\times 5}-(4\times 11)=?$
44. $(15\times 6)-\overline{7\times 12}=?$

7.

1. If a man travel 8 hours at the rate of 5 miles an hour, how far does he travel?

SOLUTION.— Since in one hour he can travel 5 miles, in 8 hours he can travel 8 times 5 miles, which are 40 miles. Therefore, he can travel 40 miles in 8 hours.

2. In one square yard there are 9 square feet; how many square feet are there in 6 square yards?

3. George is 12 years old and his father is 4 times as old; how old is his father?

4. If it require 8 tons of coal to run a train one trip, how many tons will it require to run 7 such trips?

5. How many miles will a car run in 7 hours, if it run 9 miles an hour?

6. In one peck there are 8 quarts; how many quarts in 7 pecks?

7. If a steamboat travel 11 miles an hour, how far will it travel in 11 hours?

8. How many boys in 8 base ball nines?

9. How many sides are there in 8 squares?

10. If a man put 10 dollars a month in the bank, how much will he save in one year?

11. How many minutes in 7 hours? in 9 hours?

12. How many shoes do 8 teams of horses need?

13. If a boy deliver a paper each day, how many will he deliver in 9 weeks (Sundays excepted)?

14. How far around a square that is 20 feet on a side?

15. What will 2 dozen two-cent postage stamps cost?

16. I bought a turkey weighing 12 pounds at 12 cents a pound; how many cents did it cost me?

17. How many faces have 9 cubes?

18. The Third Avenue railroad in New York city is 7 miles long; how many miles does a car run in making 7 round trips?

19. What will 12 chairs cost at $7 each?

20. If one boy earns $11, how much will 7 boys earn?

21. How many glass eyes will be needed for 19 dolls?

22. At 13 cents a yard, what will 7 yards of factory cost?

23. David has 9 cents and Jonathan has 9 times as many; how many cents has Jonathan?

24. Marion is 9 years old and her grandfather is 10 times as old; how old is her grandfather?

25. What will 8 yards of dress goods cost at 22 cents a yard?

8.

1. Jeremiah bought 2 oranges at 6 cents each, 7 lemons at 3 cents each, and 5 pears at 2 cents each; how much did the whole cost him?

SOLUTION.—The whole cost the sum of the products of 2 times 6 cents, 7 times 3 cents and 5 times 2 cents. 2 times 6 cents are 12 cents; 7 times 3 cents are 21 cents, and 5 times 2 cents are 10 cents: the sum of 12 cents and 21 cents is 33 cents, and 10 cents added equals 43 cents. Therefore, the whole cost him 43 cents.

2. A lady bought 3 yards of ribbon at 10 cents a yard, 2 spools of thread at 4 cents a spool, 6 yards of cambric at 9 cents a yard; how much money did she spend?

3. Two men start from the same place and travel in opposite directions, one at the rate of 6 miles an hour and the other at the rate of 7 miles an hour; how far apart will they be at the end of 2 hours?

4. Mr. Gregg bought 12 cords of wood at $4 a cord, and 2 loads of hay at $16 a load; what was the cost of the wood and the hay?

5. If Mr. Moore pays 2 dollars a week for 6 weeks for his room, and 5 dollars a week for the same time for his board, how much do his board and room cost him for 6 weeks?

6. A farmer sold 20 bushels of wheat at $1 a bushel and 4 dozen eggs at 12 cents per dozen; how much did he receive for the wheat and the eggs?

7. Robert bought 10 slates at 5 cents each and 12 pencils at 3 cents each; how much did his purchases cost him?

8. Find the cost of 4 pounds of sugar at 6 cents a pound, 7 bars of soap at 5 cents a bar, and 2 boxes of matches at 10 cents a box.

9. Irwin earned 3 dollars a week for 6 weeks and 5 dollars a week for 10 weeks; how much did he earn in the 16 weeks?

10. What will be the cost of 8 pounds of meat at 8 cents a pound, and 4 pounds of meat at 9 cents a pound?

11. In an orchard there are 8 rows of trees with 7 trees in a row, and 4 rows with 9 trees in a row; how many trees in the orchard?

12. Grace paid 15 cents a meal for 2 meals and Ada paid 20 cents a meal for 3 meals; how much did they both spend?

13. Find the cost of 4 yards of ribbon at 11 cents a yard, 2 yards at 14 cents a yard and 3 yards at 8 cents a yard.

14. Lucile is 6 years old and her mother is 5 times as old. If Lucile's father is 6 times her age, what is the sum of the father's age and the mother's age?

15. Mr. White paid 5 cents each for fare for 12 persons and 3 cents each for 6; how much car fare did he pay?

9.

1. Ned bought 2 plants at 10 cents each and 3 plants at 6 cents each; how much change ought he to receive from 50 cents?

ANALYSIS.—He ought to receive the difference between 50 cents and the sum of the products of 2 times 10 cents and 3 times 6 cents. 2 times 10 cents are 20 cents and 3 times 6 cents are 18 cents. The sum of 20 cents and 18 cents is 38 cents and the difference between 50 cents and 38 cents is 12 cents. Therefore, he ought to receive 12 cents change.

2. Mr. Slocum bought 3 pairs of shoes at 3 dollars a pair, 5 pairs of slippers at 2 dollars a pair, and handed the shoe dealer a twenty-dollar gold piece; how much change ought he to receive?

3. Mr. Copperfield bought for his wife 5 bird-cages at 2 dollars each, 3 fancy lamps at 8 dollars each, and a pagoda for her dog, Jip, for 5 dollars; how much less than $50 did he expend?

4. A man earned 20 dollars in 3 weeks. He spent 5 dollars a week for board, one dollar a week for his room and 2 dollars a week for books; how much did he save in 3 weeks?

5. Mr. Mills sold 3 chairs at 3 dollars each, and 4 hammocks at 2 dollars each; he took in payment 2 pairs of shoes at 2 dollars a pair and 2 pairs of slippers at one dollar a pair, and the remainder in cash; how much cash did he receive?

SOLUTION.—He received the difference between the sum of the products of 3 times 3 dollars and 4 times 2 dollars and the sum of

the products of 2 times 2 dollars and 2 times one dollar. 3 times 3 dollars are 9 dollars and 4 times 2 dollars are 8 dollars: their sum is 17 dollars. 2 times 2 dollars are 4 dollars and 2 times one dollar is 2 dollars, and their sum is 6 dollars. The difference between 17 dollars and 6 dollars is 11 dollars. Therefore, he received 11 dollars in cash.

6. James earned 10 dollars a week for 3 weeks and 12 dollars a week for 4 weeks; Edward earned 5 dollars a week for 6 weeks and 6 dollars a week for 7 weeks; which earned the greater sum and what is the difference between the amounts earned?

7. Mrs. Otis bought 6 yards of cloth at 2 dollars a yard and 3 hats at 3 dollars each; Mrs. Young bought 2 pairs of shoes at 3 dollars each and 3 dresses at 4 dollars each; how much more did Mrs. Otis spend than Mrs. Young?

8. Frank took 2 music lessons a week for 12 weeks and one a week for 40 weeks; the next year he took one a week for 30 weeks and 2 a week for the remainder of the year; how many more lessons did he take the second year than the first?

9. A man bought 6 barrels of flour at 5 dollars a barrel and sold it at a gain of 20 dollars; how much did he sell it for?

10. Henry earned 10 dollars a week and spent 5 dollars a week for board and 2 dollars a week for other expenses; how much did he save in 12 weeks?

11. Clyde has 11 dollars in the bank and his brother lacks 4 dollars of having 3 times as much; how much has his brother?

12. Bought 2 pounds of beef at 12 cents a pound, 2 bunches of radishes at 5 cents a bunch, and 2 quarts of beans at 8 cents a quart; how much did I pay for all?

10.

1. Two men starting from the same point travel in opposite directions, one at the rate of 6 miles an hour, the other at the rate of 5 miles an hour; how far apart will they be at the end of 11 hours?

SOLUTION.—Since they travel in opposite directions, they will be as far apart at the end of one hour as the sum of 6 miles and 5 miles, which is 11 miles; and in 11 hours they will be 11 times 11 miles, or 121 miles. Therefore, they will be 121 miles apart at the end of 11 hours.

2. Two men starting from the same point travel in the same direction, one at the rate of 6 miles an hour, the other at the rate of 4 miles an hour; how far apart will they be at the end of 9 hours?

NOTE.—Traveling in the *same* direction. at the end of *one hour* they will be as far apart as the *difference* between 6 miles and 4 miles, which is 2 miles.

3. Count by 2's from 0 to 24; by 3's from 0 to 36; by 4's from 1 to 49.

4. Two men travel in opposite directions, one at the rate of 7 miles an hour, the other at the rate of 5 miles an hour; how far apart will they be at the end of 4 hrs.? 6 hrs.? 12 hrs.?

5. If they travel in the same direction, how far apart will they be at the end of 5 hrs.? 7 hrs.? 9 hrs.?

6. Count by 5's from 0 to 60; from 60 to 30.

7. Count by 6's from 1 to 43; from 72 to 24.

8. If a man earned $10 a week and his son $2, how much did they both earn in 7 weeks?

9. If Mr. Smith earns $15 a week and spends $6 a week, how much can he save in 9 weeks? in 10 weeks?

10. Count by 7's from 0 to 84; by 8's from 0 to 96.

11. Count by 9's from 1 to 55; by 9's from 81 to 0.

12. Count by 10's from 0 to 120; by 11's from 0 to 132; by 12's to 144.

13. Bought 7 quarts of milk at 5 cents a quart and 6 pounds of crackers at 7 cents a pound; how much change should I receive if I give in payment a one-dollar bill?

14. If two men travel in the same direction from a certain point, one at the rate of 7 miles an hour, the other at the rate of 11 miles an hour, how far apart will they be at the end of 7 hours?

15. Henry worked 10 weeks at 9 dollars a week and Norman 7 weeks at 6 dollars a week; how much more did Henry earn than Norman?

16. A man earned 16 dollars a week for 3 weeks and his son earned 5 dollars a week for the same time; how much did they earn together?

17. George is 4 years old and Reuben is 5 times as old lacking 2 years; what is the sum of their ages?

18. Mr. Cotchefer bought 30 baskets of peaches at $2 a basket; he sold 20 baskets at $3 a basket and 10 baskets at $1 a basket; did he gain or lose on the whole and how much?

19. A gave $5 toward the Chamber of Commerce Fund for the Poor, B gave $21, C gave $19 and D gave as much as A and C together; how much did all give?

20. If a man earns 12 dollars a week and spends 5 dollars a week, how much can he save in 9 weeks?

21. If a man earns 12 dollars a week and foolishly incurs expense to the extent of 15 dollars a week, how much in debt will he find himself at the end of 5 weeks?

22. What will 6 chairs at $5 each, 2 tables at $10 each, and 2 sideboards at $25 each, cost?

11.

Practice Exercises.

1. 8×9 plus 4×7 minus 5×5 are how many?

2. 12×11 minus 4×8 plus 7×8 are how many?

3. $16 + 18, - 14, + 4, - 9, + 32$ are how many?

4. 15 plus 25 multiplied by 3 minus 7 are how many?

5. $20 - 7, + 19, + 8, \times 2, - 47, \times 2, + 18, - 6, + 19$ are how many?

6. $13 \times 3, + 24, - 13, - 28, - 11, \times 7, - 18$ are how many?

7. 12×6 minus 4×7 plus 8×9 are how many?

8. 6×7 plus 7×8 plus 9×7 are how many?

9. 9×9 minus 7×7 plus 6×4 are how many?

10. 11×11 minus $3 \times 3 \times 3$ plus 6×0 are how many?

11. 9×12 minus 2×4 plus 10×10 are how many?

12. 4×9 plus 8×3 minus 5×8 are how many?

13. How many are 5 times 16 plus 6 times 5?

14. How many are 4 times 14 minus 7 times 8?

15. How many are 8 times 6 plus 4 times 13?

16. How many are 12 times 12 minus 4 times 11?

17. How many are 5 times 15 minus 8 times 9?

18. Give the results of the following:

14×5; 16×8; 17×4; 18×4; 19×6; 15×7.

Beginning at the right of each of the following, read the results only.

Thus, in example 19, say 32, 27, 18, 9.

19.	1234	*20.*	4683	*21.*	7849	*22.*	98762
	8		4		5		6
23.	51478	*24.*	12345	*25.*	67891	*26.*	23415
	7		8		5		9
	4						

DIVISION.

Introductory Exercises.

1. How many groups of 2 marbles each can be made from 4 marbles? How many 2's in 4?

Answer. There are two 2's in 4 because 2 times 2 are 4.

2. Since there are 2 pints in one quart, how many quarts are there in 6 pints? How many times is 2 contained in 6?

Ans. 2 is contained in 6, 3 times, because 3 times 2 are 6.

3. If you pay 3 cents for an orange, how many can you buy for 9 cents? How many times are 3 cents contained in 9 cents?

4. John had 8 marbles; if he gave 2 to each member of his class, to how many boys did he give?

5. How many 2's in 8? in 10? in 12? in 14?

6. At 5 cents a quart, how many quarts of milk can be bought for 15 cents? How many 5's in 15?

Definitions.

1. Division is the process of finding how many times one number is contained in another. It is the process of separating a number into equal parts. *Division is a short method of subtracting* when the numbers to be subtracted are equal.

2. The Dividend is the number to be divided.

3. The Divisor is the number by which to divide.

4. The Quotient is the number found by dividing.

5. The Remainder is the part of the dividend left over when the division is not exact. If there is a remainder after dividing any figure or figures of the dividend,

regard it as prefixed to the next term of the dividend and continue the division. When there is a final remainder, consider it, with the divisor beneath, a part of the quotient.

6. The SIGN of DIVISION is ÷. It is read *divided by.* When placed between two numbers, it shows that the number on the left is to be divided by the one on the right.

Thus, 12 ÷ 3 is read 12 divided by 3.

7. In the expression, 12 ÷ 3 = 4, the dividend is 12, the divisor 3, and the quotient 4.

In the expression, 12 ÷ 5 = $2\frac{2}{5}$ (read two and two-fifths), the dividend is 12, the divisor 5, the quotient 2, and the remainder 2. The remainder is written over the divisor as part of the quotient.

8. Division is also expressed by writing the dividend above and the divisor below, a short horizontal line.

Thus, $\frac{15}{3} = 5$, shows that 15 divided by 3 equals 5. This is sometimes called a *Fractional Sign.*

9. To find the *Dividend,* multiply the Divisor and Quotient together and to the product add the Remainder, if any.

10. To find the *Divisor,* subtract the Remainder, if any, from the Dividend and divide the result by the Quotient.

1.

PROBLEMS.

1. At $2 a basket, how many baskets of peaches can be bought for $12?

SOLUTION.—Since one basket cost $2, as many baskets can be bought for $12 as $2 is contained times in $12, which is 6 times, or 6 baskets. Therefore, 6 baskets of peaches at $2 a basket can be bought for $12.

2. If one book cost $2, how many books can be bought for $14?

3. Edward gave 15 cents for 3 bottles of ink; how much was that per bottle?

4. At 3 cents a yard, how many yards of ribbon can be bought for 18 cents?

5. Mary divided 21 cents among her three sisters; how many cents did she give to each?

6. I have 24 oranges; how many children can I supply if I give 3 oranges to each child?

8. If 4 horses draw 8 tons of hay, how many tons can one horse draw?

8. If the toll for 4 horses on the East Side boulevard is 40 cents, what is the toll for one horse?

9. How many quarts of salt at 4 cents a quart can be bought for 36 cents?

10. How many spools of thread at 4 cents each can be bought for 48 cents?

Practice Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
11. Divide	8	18	21	16	24	9	27	15	36	12
by	2	3	3	2	2	3	3	3	3	2
12. Divide	3	16	16	32	8	20	6	12	24	16
by	3	4	2	4	4	4	3	3	4	4

13. If one ride in an electric car cost 5 cents, how many rides can John get for 25 cents?

14. How many bars of soap at 5 cents a bar can I buy for 35 cents?

15. Julia divided 30 peaches equally among 5 girls; how many did she give to each?

16. If Edwin earns $5 a week, how long will it take him to earn $40?

17. How many boys will it take to sharpen 50 pencils if each boy sharpens 5?

18. How many tops can you buy for 12 cents, at 6 cents each?

19. How many verbenas at 6 cents each can you buy for 24 cents?

20. At 6 cents a yard, how many yards of factory can be bought for 54 cents?

21. How many fathoms in 30 feet, if in one fathom there are 6 feet?

22. If Mr. Brown paid $56 for 7 books, how much did he pay for one book?

23. What will one coat cost, if 7 coats cost $63?

24. If you have 28 cents to be spent for grapes, how many pounds at 7 cents each can you buy?

25. If one man can do a piece of work in 35 days, how long will it take 7 men to do it?

26. Divide each of the following numbers by 5: 10, 60, 15, 25, 45, 40, 20, 30, 50, 35.

27. Divide each of the following numbers by 6: 6, 18, 42, 54, 30, 24, 60, 72, 66, 48, 36, 12.

28. Divide each of the following numbers by 7: 14, 28, 42, 49, 21, 63, 70, 84, 35, 56, 77, 7.

29. If 8 girls earned $32, how much did one girl earn?

30. How many pecks in 40 quarts, if in one peck there are 8 quarts?

31. If 48 pairs of stockings are to be divided equally among 8 men, how many pairs will each receive?

32. If a ship can sail 8 miles in 56 hours, how far can she sail in one hour?

33. If I have 24 pages to copy, how long will it take me at the rate of 8 pages a day?

34. At 8 cents a yard, how many yards of ribbon can be bought for 96 cents?

35. At 9 cents each, how many slates can I buy for 27 cents?

36. If 9 bars of cocoanut candy cost 45 cents, what will one bar cost?

37. How many pounds of rice at 6 cents a pound can be bought for 54 cents?

38. If one coach will hold 9 passengers, how many coaches will it take to hold 63 passengers?

39. How much lining at 9 cents a yard can I buy for 72 cents?

40. How many dishes of ice cream at 10 cents a dish can be bought for 40 cents?

41. How long will it take me to work 50 examples at the rate of 10 examples an hour?

42. How many dollars in 100 dimes, if in one dollar there are 10 dimes?

43. If one fruit jar cost me 10 cents, how many can I buy for 110 cents?

44. Divide each of the following by 8: 8, 32, 40, 56, 72, 88, 96, 80, 64, 48, 24, 16.

45. Divide each of the following by 9: 18, 36, 54, 63, 81, 9, 45, 108, 72, 90, 99, 27.

46. Divide each of the following by 10: 10, 100, 120, 110, 20, 50, 30, 40, 70, 60, 80.

47. At $11 apiece, how many chairs can I buy for $66?

48. How many rows will it take to seat 77 persons, if each row will seat 11 persons?

49. At $11 each, how many coats can I buy for $99?

50. At $11 each, how many books can be bought for $132?

51. What is the cost of one sandwich if 11 sandwiches cost 55 cents?

52. If I save $11 a month, how long will it take me to save $88?

53. If in one dozen there are 12 eggs, how many dozen in 48 eggs?

54. How many boys carrying 12 pounds each will be needed to carry 60 pounds?

55. At $12 each, how many books can be bought for $72?

56. At 12 cents a pound, how many pounds of raisins can I buy for 84 cents?

57. How many dozen bananas at 12 cents a dozen can be bought for 96 cents?

58. If a man earn $12 a week, how long will it take him to earn $144?

59. Divide each of the following by 11: 33, 55, 99, 110, 44, 22, 11, 77, 121, 88, 132, 66.

60. Divide each of the following by 12: 120, 84, 72, 60, 96, 48, 36, 24, 144, 132.

Practice Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
61. Divide	27	36	18	45	28	30	54	24	35	12
by	3	3	2	5	4	5	6	3	5	4
62. Divide	25	24	45	54	30	48	14	28	36	21
by	5	6	5	6	5	6	7	7	6	7
63. Divide	70	32	84	40	18	24	63	16	7	8
by	7	8	7	8	9	8	7	8	7	8

64. Divide	36	70	33	36	77	45	80	84	121	81
by	9	10	11	12	11	9	10	12	11	9

2.

Equal Parts.

1. When anything is divided into two equal parts, each part is called *one-half.* Hence, one-half of anything is one of the *two* equal parts into which it can be divided, and is written $\frac{1}{2}$.

2. When anything is divided into three equal parts, each part is called *one-third.* Hence, one-third of anything is one of the *three* equal parts into which it can be divided, and is written $\frac{1}{3}$.

3. When anything is divided into four equal parts, each part is called *one-fourth.* Hence, one-fourth of anything is one of the *four* equal parts into which it can be divided, and is written $\frac{1}{4}$.

4. When anything is divided into five equal parts, each part is called *one-fifth* and is written $\frac{1}{5}$. When into six equal parts, each part is called *one-sixth,* and is written $\frac{1}{6}$, and so on. The name is taken from the number of equal parts into which the whole thing is divided.

3.

1. What is $\frac{1}{2}$ of 4?

Solution 1.—It is that number which taken 2 times will make 4. 2 times 2 are 4. Therefore, $\frac{1}{2}$ of 4 is 2. (or)

Solution 2.—If 4 units are divided into two equal parts, each part will contain 2 units. Therefore, $\frac{1}{2}$ of 4 is 2.

2. What is one-third of 12 dollars? $15? $21?

3. What is one-fourth of 16 bushels? 20 bu.? 32 bu.?

4. What is one-fifth of 20 pecks? 60 pks.? 90 pks.?

5. What is one-sixth of 36 cents? 54¢? 42¢?

6. What is one-seventh of 49? 56? 84?

7. What is one-eighth of 64? 72? 96?

8. What is one-ninth of 81 dollars? $36? $27?

9. What is one-tenth of 100 men? 120 men? 30 men?

10. What is one-eleventh of 121 ounces? 88 oz.? 66 oz.?

11. What is one-twelfth of 72 pencils? 60 days? 84 hours?

12. If 6 pounds of sugar cost 30 cents, what will one pound cost?

SOLUTION.—If 6 pounds of sugar cost 30 cents, one pound will cost $\frac{1}{6}$ of 30 cents, or 5 cents. Therefore, one pound will cost 5 cents.

13. If we divide 30 oranges equally among 6 people, how many oranges will each receive?

14. If 7 tons of coal cost $42, what will one ton cost?

15. If 8 pencils cost 32 cents, what will one pencil cost?

16. If 3 hats cost $9, what will one hat cost?

17. If 4 horses eat 28 quarts of oats, how many quarts will one horse eat?

18. If 8 yards of ribbon cost 56 cents, what will one yard cost?

4.

1. If 7 yards of cloth cost 14 dollars, what will five yards cost?

SOLUTION.—If 7 yards cost $14, one yard will cost $\frac{1}{7}$ of 14 dollars, which is 2 dollars; since 1 yard cost 2 dollars, 5 yards will cost 5 times 2 dollars, or 10 dollars. Therefore, 5 yards of cloth will cost 10 dollars.

NOTE.—In analysis we proceed from the given number to *unity* or *one*, and from unity or one to the *required* number. The unit (or 1) being the basis of the reasoning process.

Thus, in the above problem, we reasoned from the cost of *7* yards to the cost of *1* yard, and from the cost of *one* yard to the cost of *five* yards, the required number.

2. How much will 12 sheets of paper cost, if 11 sheets cost 22¢?

3. If 8 yards of cloth cost $56, what will 5 yards cost?

4. If Mary can do 8 examples in 48 minutes, how long will it take her to do 10 examples?

5. If the rent of a house for 9 weeks is $54, what is the rent for 7 weeks?

6. If 5 gallons of milk cost 75 cents, what will 9 gallons cost?

7. If 6 gas stoves cost 90 dollars, what will 10 cost?

8. If 8 fur capes cost 160 dollars, what will 12 fur capes cost?

9. If 9 spools of silk cost 72 cents, what will 25 spools cost?

10. If 6 crocks of butter cost $24, what will 9 crocks cost?

5.

1. How many apples at 2 cents each must you give for 3 oranges at 4 cents each?

SOLUTION.—You must give as many apples at 2 cents each as 2 cents is contained times in the product of 3 times 4 cents. 3 times 4 cents are 12 cents, and 2 cents is contained times in 12 cents 6 times, or 6 apples. Therefore, you must give 6 apples for 3 oranges at 4 cents each.

2. How many barrels of flour at 6 dollars a barrel will pay for 9 yards of cloth at 4 dollars a yard?

3. How many quarts of berries at 8 cents a quart will pay for 12 bars of soap at 10 cents a bar?

4. How many yards of ribbon at 12 cents a yard will pay for 9 yards of ribbon at 16 cents a yard?

5. If I exchanged 4 dozen eggs for 8 dozen lemons at 12 cents a dozen, what was the value of one dozen eggs?

6. How many books at 5 dollars each will be equivalent in value to 10 books at 7 dollars each?

7. If 6 men can do a piece of work in 8 days, how long will it take 8 men to do the same work?

8. How many gallons of gasoline at 12 cents a gallon will be equivalent in value to 9 pounds of butter at 20 cents a pound?

9. How many men working 8 hours a day can accomplish the same work that 16 men can do working 10 hours a day?

10. How many tons of coal at 6 dollars a ton will be equivalent in value to 7 tons of hay at 12 dollars a ton?

11. How many bathing suits, each containing 5 yards, can be made from 10 pieces of cloth, each containing 12 yards?

12. How many Long Branch rockers at 2 dollars each will pay for 7 couches at 14 dollars apiece?

13. At 7 dollars each, how many sofa pillows will be equivalent in value to 21 yards of silk at 2 dollars a yard?

14. If I exchanged 6 handkerchiefs for 9 towels at 24 cents each, what was the value of one handkerchief?

15. Mr. Stiles gave 5 pictures for 10 days' labor at 3 dollars a day; what was the value of one picture?

16. Miss Gold exchanged 11 yards of dress goods for 22 pairs of shoes at 4 dollars a pair; what did she receive for one yard of cloth?

17. In exchange for 9 weeks' rent, a man worked 3 weeks at 12 dollars a week; how much did he pay for one week's rent?

18. Divide the product of 9 times 32 by 8.

6.

1. How many apples must you give for 9 peaches at 2 cents each, if 4 apples are worth 8 cents?

SOLUTION.—If 4 apples are worth 8 cents, one apple will be worth $\frac{1}{4}$ of 8 cents, or 2 cents. If one peach is worth 2 cents, 9 peaches will be worth 9 times 2 cents, or 18 cents. You can buy as many apples for 18 cents as 2 cents is contained times in 18 cents, which is 9 times or 9 apples. Therefore, 9 apples must be given for 9 peaches at 2 cents each.

2. If 6 tons of coal cost $30, how many tons must be given for 9 clocks at $5 each?

3. If 4 dozen of eggs cost 40¢, how many dozen must be given for 10 pounds of sugar at 6¢ a pound?

4. How many plants can I get for 12 boxes of cherries at 5¢ a box, if 7 plants cost 70¢?

5. Mr. Gillette bought 2 baskets of peaches at $2 each, 6 baskets of pears at $3 each, and 2 boxes of soap at $2 each; how many deeds must he make out in order to pay for his services if he makes out 6 deeds for $18?

6. If Mr. Williams sold 3 tickets for $15, what sum should he receive for 7 tickets at the same rate?

7. How many pounds of crackers must be given for 12 pounds of sugar at 5¢ a pound, if 7 pounds of crackers are worth 42¢?

8. How many pounds of sweet potatoes are equivalent in value to 3 pounds of coffee at 30¢ a pound, if 6 pounds of potatoes cost 30¢?

9. If 4 pounds of butter are worth 80 cents, how many pounds of butter must be given for 10 pounds of meat at 12 cents a pound?

10. If 7 lead pencils cost 35 cents, how many pencils must be given in exchange for 8 books worth 40 cents each?

11. If 9 boxes of hairpins cost 108 cents, how many boxes will be equivalent in value to 6 yards of ribbon at 10 cents a yard?

12. How many two-cent stamps are equivalent in value to nine six-cent stamps?

13. If 11 bottles of ink cost 99 cents, how many bottles must be given for 3 souvenir pins worth 66 cents each?

14. How many pairs of shoes at the rate of 6 pairs for 24 dollars must be given for 3 tons of hay at 16 dollars a ton?

15. If 10 gold rings are worth 120 dollars, how many must be given for 3 gold watches worth 72 dollars each?

16. If 9 yards of cloth cost $18, how many yards are equivalent in value to 5 couches worth 28 dollars each?

17. If 6 tables are worth 72 dollars, how many tables are equivalent in value to 12 chairs worth 4 dollars each?

7.

1. Give the quotient of 20, 5, 45, 10, 65 divided by 5.

2. Give the quotient of 25, 40, 30, 15, 50, 35 divided by 5.

3. Give the quotient of 60, 36, 66, 24, 72, 18 divided by 6.

4. Give the quotient of 21, 49, 14, 63, 35, 42 divided by 7.

5. Give the quotient of 72, 32, 88, 56, 96, 16 divided by 8.

6. Give the quotient of 18, 72, 54, 81, 45, 27 divided by 9.

7. Give the quotient of 22, 55, 121, 110, 44, 132 divided by 11.

8. Give the quotient of 24, 72, 60, 36, 84, 144 divided by 12.

9. Give the quotient and remainder of the following: 45, 56, 59, 41, 50, 51, 27, 46, 53, 39, 17, 23 divided by 6.

Thus, in the number 45, say 7, 3.

NOTE.—When there is a remainder after dividing any figure or figures of the dividend, regard it as prefixed to the next term of the dividend and continue the division. When there is a final remainder, consider it, with the divisor beneath, a part of the quotient.

Thus, in 97 divided by 4.

4 is contained in 97, 24¼ times. (Read twenty-four and one-fourth.)

8.

1. What is the quotient of 45 divided by 3?
2. What is the quotient of 57 divided by 4?
3. What is the quotient of 126 divided by 5?
4. What is the quotient of 216 divided by 6?
5. What is the quotient of 98 divided by 7?
6. What is the quotient of 99 divided by 8?
7. What is the quotient of 111 divided by 9?
8. What is the quotient of 142 divided by 10?

9.

1. 9 times 5 plus 15 are how many times 12? 15? 7?

Solution.—9 times 5 plus 15 are as many times 12 as 12 is contained times in the sum of 15 and the product of 9 times 5. 9 times 5 are 45; the sum of 45 and 15 is 60, and 12 is contained in 60, five times. Therefore, etc.

2. $14 \times 4 + 8$ are how many times 8? 12? 5?

3. $6 \times 7 + 3$ are how many times 9? 5? 15?

4. $9 \times 9 - 1$ are how many times 10? 8? 5?

5. $6 + (8 \times 9)$ are how many times 8? 9? 10?

6. $54 - (3 \times 2)$ are how many times 2? 3? 4?

7. 6 times 5 plus 3 are how many times 5? 6? 7?

8. Name two sets of two numbers each whose product is equal to the following:

12, 30, 16, 36, 40, 84, 63, 96, 88, 80, 99, 100.

Find the value of the following:

9. $8 \times 5, + 2, \div 7, \times 6, + 4, \div 10, \times 14, + 6, + 9, - 8.$

10. $16 + 17, - 18, \times 6, + 1, \div 13, \times 14, + 2, \div 10, + 25.$

11. $19 + 11, + 13, + 2, + 16, + 5, + 14, + 18, + 4, + 8, + 3, + 10.$

12. $98 \div 7, + 7, + 8, + 7, \div 12, \times 29, + 3, + 12, + 30, \div 12.$

13. Multiply 8 by 4, subtract 2, divide by 5, add 4, divide by 2, multiply by 7, add 1, divide by 9; what is the result?

14. Multiply 6 by 4, add 12, divide by 9, add 6, multiply by 3, subtract 3, divide by 9; what is the result?

10.

General Review.

1. If May is 10 years old and John is 9 years older, what is the sum of their ages?

2. Frank paid 17¢ for a top and sold it for 8¢; how much did he lose?

3. James lives 13 rods east of the school entrance and John 5 rods west; how much farther does James live from the school house than John? How far apart do they live?

4. Two girls walk from the same place. One goes up street, the other down street. Each walk 11 yards. How far apart are they?

5. Walter is 22 years old. How old was he 8 years ago?

6. How much change should Ida receive from $20, if her dress cost $12?

7. A man having 20 tubs of butter, sold 3 at one time, 5 at another, and 6 at another; how many tubs had he left?

8. From a pole 24 feet long, 9 feet were cut at one time, 6 at another, and 3 at another. How many feet were cut off? How many remained?

9. Martha bought 8 cents worth of candy, 6 cents worth of ribbon, and 2 two-cent postage stamps. She gave the clerk a 25-cent piece in payment. How much change should he give her?

10. Augusta was 5 years of age 4 years ago; how old is she now? How old will she be in 8 years?

11. If the sum of two numbers is 18 and one of the numbers is 9, what is the other number?

12. Jane is 7 years old and her sister is 8 years older. How old is her sister? What is the sum of their ages?

13. At one time our flag had 13 stars; how many have been added to make the number 44?

14. Two men start from the same point and travel east, one at the rate of 32 miles an hour; the other at

the rate of 27 miles an hour; how far apart are they at the end of one hour?

15. During a warm day the thermometer registered 96 degrees; how many degrees was that above freezing, which is 32 degrees?

16. There are 41 pencils on my desk; if 26 of them are slate pencils and the rest are lead pencils, how many are lead pencils?

17. If Mrs. Fish borrowed $80 of a friend and paid back $37, how much does she still owe?

18. When a man was 39 years old, he had been married 15 years; how old was he when he was married?

19. If 6 books cost $30, how many can you buy for $45?

20. If a man can build 72 rods of fence in 8 days, how many rods can he build in 10 days?

21. If 9 dozen eggs cost 108¢, what will 6 dozen cost?

22. Mr. Smith gave $26 to his four sons; to one he gave $5, and the remainder he divided equally among the other three; how much did each receive?

23. A pansy has 5 petals; how many petals have 16 pansies?

24. Charles worked 10 months for 12 dollars a month, and then spent 100 dollars for a bicycle; how much had he left? How many times can he take his bicycle to the "Bicycle Hospital" if each visit there costs him 4 dollars?

25. The sum of three numbers is 30; two of the numbers are 18 and 7; what is the other number?

26. Three men, A, B and C, had 42 sheep; A had 19 of them, B 8 of them, and C the remainder; how many sheep had C?

27. A drover bought 12 sheep at $7 a head, and sold them for $96; how much did he gain a head?

28. Arthur is 6 years old, Charles is 14, and the sum of Arthur's age, Charlie's age and Walter's age is 35 years; how old is Walter?

29. Mr. Wallace bought a horse for $75, a harness for $20, and sold the whole for $99; how much did he gain?

30. The material for a dress cost $20 and $7 was paid for having it made; how much was gained by selling the dress, when made, for $32?

31. Cassius found 9 eggs in one nest, 7 in another, and 11 in another; how many must the fourth nest contain to make the whole number of eggs 36?

32. A woman paid 25 dollars for a sewing machine, 2 dollars for express charges and 1 dollar for cartage. She sold it for 40 dollars; how much did she gain?

33. If William paid 7 dollars for a book, 4 dollars for a hat and 2 dollars for gloves, how much had he left from a twenty-dollar bill?

34. In four weeks John sold 54 bushels of apples; the first week he sold 6 bushels, the second 9 bushels, and the third week 12 bushels; how many bushels did he sell the fourth week?

35. It cost me $15 for the different numbers of a book and $12 for the binding. A friend paid me $20 for the books and I paid $1 for their safe delivery; how much did I lose?

36. A horse traveled 15 miles one day and 25 miles the next day; on the third day he traveled 6 miles on his return and on the fourth day 15 miles on his return; how far was he from the place of starting?

37. Edward earned $16 one month and $15 the next month. He spent $5 the first month and $8 the next month; how much did he save?

38. What is the difference between the sum of 16 dollars and 24 dollars, and the sum of 9 dollars and 18 dollars?

39. Ethel worked 12 examples one day and 18 the next; if she missed 2 of them one day and 3 the next day, how many of the whole number were correct?

40. Mr. Moore bought a horse for 70 dollars and a saddle for 14 dollars; he sold the horse for 65 dollars and the saddle for 17 dollars; did he gain or lose on the sale of both, and how much?

41. Samuel earned 13 dollars, Ned earned 22 dollars, and Charles earned 31 dollars; Cora earned 16 dollars, Mary earned 20 dollars, and Enid earned 29 dollars; which earned the greater sum, the boys or the girls, and how much?

42. Mr. Drake earns $90 a month. He pays $20 a month rent, $15 for groceries and vegetables, $15 for meat, $25 for clothing and incidentals; how much does he save in one week?

43. A man bought 3 pounds of raisins at 7 cents a pound, 16 oranges at 4 cents each, 3 pounds of candles at 12 cents a pound, and gave in payment 11 dozen of eggs; how much were the eggs per dozen?

44. Two men start from the same point and travel in the same direction, one at the rate of 6 miles an hour, the other at the rate of 5 miles an hour; how far apart will they be at the end of 7 hours?

45. Two men start from the same point and travel in opposite directions, one at the rate of 4 miles an hour, the other at the rate of 3 miles an hour; how far apart are they at the end of 5 hours?

46. Ned had $39; he kept $7, and divided the rest equally among 8 friends; how many did each receive?

47. A fox is 80 rods in advance of a hound, and is running at the rate of 27 rods in a minute; the hound is following at the rate of 31 rods in a minute; in how many minutes will the hound overtake the fox?

48. If 6 men can do a piece of work in 9 days, in how many days can 2 men do it?

49. If 9 men can do a piece of work in 4 days, in what time can 6 men do a piece of work twice as large?

50. If 8 sheep cost me $32, what will 11 sheep cost?

51. From 3 pieces of broadcloth, each containing 12 yards, two patterns, one containing 10 yards and the other 8 yards, were cut; how many yards were left?

52. Mr. Stewart paid $45 for 9 weeks' board; how much should he pay for 7 weeks' board?

53. A has 6 dollars, B has 4 times as much as A, lacking 2 dollars, and C has as much as the other two together; how much money have A, B and C together?

54. How many quarts in 108 pints?

55. I bought a cow for 30 dollars and a horse for 70 dollars; if I paid 50 dollars, how much do I still owe?

56. When vinegar is 5 cents a quart, how many gallons can you buy for 60 cents?

57. A groceryman bought 20 quarts of milk; he sold at one time 10 quarts, at another 3 quarts, and at another 2 quarts; how many pints had he left?

58. Find the cost of 1 gallon and 3 quarts of syrup at 20 cents a quart.

59. How much can a man save in 5 weeks, if he earns $12 a week and pays $8 a week for board and other expenses?

60. How many cups, holding a pint each, can be filled from a three-gallon pail?

61. A man put $92 in the bank; he drew out at one time $19, at another $24, and at another $37; how many dollars had he left in the bank?

62. Christmas, 1893, Mr. Irvin spent $60; he bought 4 presents costing $3 each for his sisters, 5 presents costing $4 each for his nephews, 2 presents costing $2 each for his brothers, and divided the rest of his money equally upon a present for his mother and one for his father; how much did he spend for each of his parents' presents?

63. Fred has 3 five-cent pieces and a quarter of a dollar; if he spends $\frac{1}{5}$ of it, how much will he have left?

64. One quarter and four nickels make how many cents?

65. A man had $50; he spent $20, and then earned enough to buy 8 yards of cloth at $7 a yard by using all his money; how much did he earn?

66. If 4 gentlemen and 5 ladies go on an excursion, and the expenses, which are $4 for each person, are paid by the gentlemen, how much must each gentleman pay?

67. The greater of two numbers is 3 times the less, and the sum of the numbers is 36; what are the numbers?

SOLUTION.—The smaller is one part and the larger is 3 parts; hence, the sum is 4 parts or the whole number, which is 36. If 4 parts are equal to 36, one part is equal to $\frac{1}{4}$ of 36, or 9, the smaller number. 3 times 9, or 27, is the greater part. Therefore, the numbers are 9 and 27.

68. The sum of two numbers is 60, and the greater is 4 times the less; what are the numbers?

69. The sum of two numbers is 90, and the greater is 5 times the less; find the numbers.

70. The difference between two numbers is 35, and the greater is 6 times the less; what are the numbers?

SOLUTION.—The smaller is one part and the larger is 6 parts; hence, the difference is 5 parts, or 35; if 5 parts equal 35, one part equals $\frac{1}{5}$ of 35, or 7, the smaller part. 6 times 7, or 42, the greater part. Therefore, the numbers are 7 and 42.

71. The difference between two numbers is 44, and the greater is 5 times the less; what are the numbers?

72. The difference between two numbers is 52, and the greater is 5 times the less; find the numbers.

73. The sum of two numbers is 144, and the greater is 11 times the less; what are the numbers?

74. The sum of two numbers is 46, and the greater is 3 times the less minus 10; what are the numbers?

SOLUTION.—Since the greater is 3 times the less minus 10, we add 10 to 46 to have a number 56, that the greater shall be just 3 times the less. The less is one part, the greater 3 parts, and the sum of the two is 4 parts, or the whole number, 56. If 4 parts of a number are 56, one part is equal to $\frac{1}{4}$ of 56, or 14, the less number, and 3 times 14, or 42, the greater. But, as we added 10 to make the greater 3 times the less, we must now subtract 10 from 42 and have 32, the greater. Therefore, the numbers are 14 and 32.

75. The sum of two numbers is 42, and the greater is 3 times the less minus 10; what are the numbers?

76. The sum of two numbers is 59, and the greater is 6 times the less minus 4; what are the numbers?

77. The difference between two numbers is 32, and the greater is 3 times the less minus 4; what are the numbers?

78. The sum of two numbers is 25, and the greater is 3 times the less plus 1; what are the numbers?

79. The difference between two numbers is 43, and the greater is 8 times the less plus 1; what are the numbers?

80. The sum of two numbers is 68, and the greater is 6 times the less minus 9; what are the numbers?

81. If 6 oranges cost 24 cents, what will 12 oranges cost?

NOTE.—When the required number is an exact number of times a given number, reason as follows:

SOLUTION.—If 6 oranges cost 24 cents, 12 oranges, which are 2 times 6 oranges, will cost 2 times 24 cents, or 48 cents. Therefore, 12 oranges will cost 48 cents, if 6 oranges cost 24 cents.

82. If 4 pounds of cheese cost 60 cents, what will 8 pounds cost?

83. If 3 yards of cloth cost 15 cents, what will 12 yards cost?

84. What will 9 dozen eggs cost if 3 dozen cost 90 cents?

85. What will 8 yards of ribbon cost if 2 yards cost 20 cents?

86. If 6 pocket-stoves cost $12, what will 12 pocket-stoves cost?

87. What will 15 eggs cost at the rate of 3 eggs for 5 cents?

SOLUTION.—If 3 eggs cost 5 cents, 15 eggs, which are 5 times 3 eggs, will cost 5 times 5 cents, or 25 cents. Therefore, 15 eggs will cost 25 cents, if 3 eggs cost 5 cents.

88. What will 18 oranges cost at the rate of 2 oranges for 7 cents?

89. What is the cost of 48 lemons at the rate of 4 for 9 cents?

90. What will 24 packages of Pepsin gum cost at the rate of 6 packages for 5 cents?

91. What will 2 dozen fish-hooks cost at the rate of 12 for 18 cents?

92. What is the cost of 72 iron bolts at the rate of 8 for 10 cents?

93. How long will it take a bicycle rider to travel 25 miles at the rate of 5 miles in 12 minutes?

94. A fruit dealer sold 2 dozen oranges at the rate of 4 for 10 cents, and with the money bought peanuts at 5 cents a quart; how many quarts did he buy?

95. What number added to 4 times 7 equals 63?

96. What three numbers besides 10 will divide 120 without a remainder?

97. If 5 men can build a wall in 9 days, in how many days can 3 men build it?

SOLUTION.—If 5 men can build it in 9 days, one man can build it in 5 times 9 days, or 45 days; and 3 men can build it in $\frac{1}{3}$ of 45 days, or 15 days. Therefore, 3 men can build it in 15 days.

98. How long will it take 9 men to do the same work that 12 men can do in 3 days?

99. How long will it take 7 men to do the same work that 4 men can do in 14 days?

100. The sum of two numbers is 21, and the greater 11; what is the product of the numbers?

12.

1. What number is that to which if 25 be added, and the sum multiplied by 9, the product will be 504?

EXPLANATION.—We divide 504 by 9 to find what number multiplied by 9 gave the product 504. Dividing, we find 56 to be that number. We then subtract 25 from 56 to find to what number 25 was added to make 56. Subtracting, we find 31, which is the number that answers the conditions.

2. What number, diminished by 40, and the remainder divided by 5, will give the quotient 58?

3. What number, increased by 16, and the sum divided by 9, will give 24 for a quotient?

4. What number is that to which, if 8 be added, the sum divided by 8, 3 subtracted from the quotient, the remainder multiplied by 14, and 12 added to the product, the result will be 54?

5. A certain number, diminished by 3, the remainder divided by 9, the quotient multiplied by 5, the product increased by 16, gives 31 for the result; what is the number?

6. What number, divided by 3, the quotient increased by 9, the sum multiplied by 2, that product divided by 7, the quotient increased by 16, equals 20?

13.

1. Add 8 to 10, divide by 6, multiply by 9, subtract 7, multiply by 5, add 20, divide by 10, add 8, and what will be the result?

2. Multiply 8 by 3, add 1, divide by 5, multiply by 2, add 2, divide by 3, multiply by 10; what is the result?

3. Six plus 12, divided by 9, multiplied by 7, minus 4, plus 11, divided by 7, equals what?

4. Divide 40 by 8, multiply the quotient by 9, to the product add 11, from the sum take 6, and what will remain?

5. Subtract 10 from 6 times 8, to the remainder add 4, divide the sum by 6, multiply the quotient by 11, subtract 7 from the product, divide the remainder by 10, add 30 to the quotient, and what will be the sum?

6. $(8 \times 9) \div 12 + (3 \times 10) - (4 \times 5), \div (4 \times 4) =$ what?

7. $(7 \times 7) - (6 \times 4), \div 5, + (3 \times 4) + (8 \div 4) =$ what?

8. $9 + 4 - 4 - 1 + 3 + 2 \div 15 \times 5 =$ what?

9. $42 - (3 \times 10), \div 3, + (5 \times 5), \div 3 =$ how many?

10. $(3+3+5)-(7+4)+(6\times3)\div(3\times3)=$how many?

11. $15\div5\times9,-7,+10,-(120\div12)\div(36\div9)=$what?

12. $(4\times4)-(3\times3)+(49\div7)+(56\div4)=$ what?

13. $(8\times9)-\frac{49}{7}+\frac{16}{4}+(13\div13)=$ how many?

14. $(5+6)-(3\times2)+(25\div5)=$ how many?

15. $(7\times3)-(14\div7)+(30\div6)=$ how many?

16. $\frac{88}{11}+\frac{132}{12}-\frac{56}{7}+\frac{48}{4}-\frac{96}{8}=$ how many?

17. $(6\times7)+(96\div12)-(6\times5)+(55\div11)=$ what?

18. $\frac{24}{6}+8+6\div(7-4)-(9\div3)=$ how many?

19. $5\times6-\frac{36}{12}+(7+2)+(\ )=44.$

20. $16\div4,\times8,-9,+12,\div(\ )=7.$

21. $\frac{30}{3}-\frac{40}{8}+\frac{65}{5}-\frac{18}{6}+(-)=25.$

22. $3+8-4-(\frac{12}{3}-2)+(\frac{36}{4}\div\frac{9}{3})=$ how many?

23. $12\times6\div8-(15\div5)+\frac{18}{2}-\frac{30}{6}=$ how many?

24. $(4\times7)-72\div9+(8+2)-(\ )=16.$

25. $(1\times2\times3\times0)+(2\times3\times4)-\frac{144}{12}=$ how many?

26. $(2\times3\times4)-(0\times1\times2\times3)-\frac{144}{12}=$ how many?

27. $1+2+3+4+5+6+8+9=\frac{0}{2}.$

28. $6\times5-(4\times7)\div\frac{16}{8}\times(8\times9)=$ how many?

29. $(7\times7)+(121\div11)\div(72\div12)=$ how many?

30. $\frac{132}{12}+\frac{66}{6}-\frac{15}{3}+\frac{42}{7}=$ how many?

31. $(8\times7\div4)-(9\times4\div6)+(3\times7)=$ how many?

32. $(5\times5)-(4\times3)-(26\div2)\times(6\div2)=$ how many?

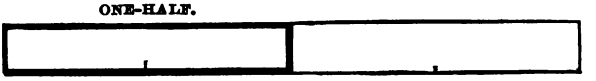
33. $(4\times3\times3\times2)-(5\times6\times2)\times(144\div12)=$how many?

FRACTIONS.

INTRODUCTION.

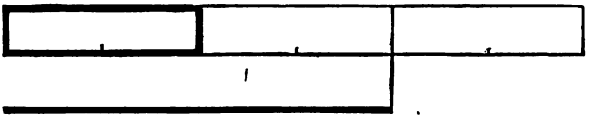
1. When anything is divided into *two* equal parts, what is each part called?

ONE-HALF.



2. When anything is divided into *three* equal parts, what is each part called? What are two parts called?

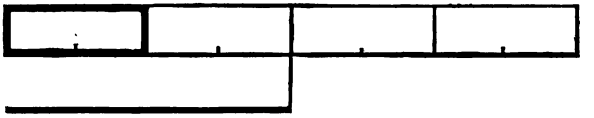
ONE-THIRD.



TWO-THIRDS OR FOUR-SIXTHS.

3. When anything is divided into *four* equal parts, what is each part called? What are two parts called?

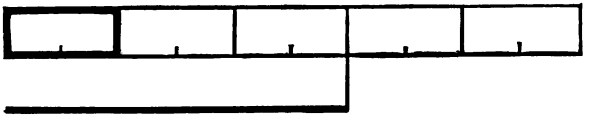
ONE-FOURTH OR ONE QUARTER.



TWO-FOURTHS, FOUR-EIGHTHS OR ONE-HALF.

4. When anything is divided into *five* equal parts, what is each part called? What are two parts called?

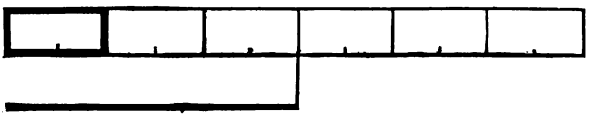
ONE-FIFTH.



THREE-FIFTHS OR SIX-TENTHS.

5. When anything is divided into *six* equal parts, what is each part called? What are three parts called?

ONE-SIXTH.



THREE-SIXTHS, OR SIX-TWELFTHS, OR ONE-HALF.

6. We may get sevenths, eighths, ninths, tenths, etc., in the same manner by dividing a whole thing into seven, eight, nine, ten, etc., equal parts. The name is taken from the number of equal parts into which the whole thing is divided.

7. By referring to the above diagrams, one-half is equal to how many fourths? By dividing each third above into two equal parts, what is each part called? How many parts in the whole? How many of these parts are required to equal one-half?

8. By dividing each fourth into two equal parts, what is each part called? How many of these parts are required to equal the whole? to equal one-half?

9. By dividing each fifth into two equal parts, what is each part called? How many of these parts are required to equal the whole?

10. By dividing each sixth into two equal parts, what is each part called? How many of these parts are required to equal the whole? to equal one-half?

11. The *greater* the *number* of parts into which the whole thing is divided, the *smaller* each part must be. One-half is twice as large as one-fourth, three times as large as one-sixth, four times as large as one-eighth, etc. Hence,—

12. The GREATER *the number of equal parts into which the* THING *is divided, the* LESS *will be the value of each part. The* FEWER *the number of equal parts, the* GREATER *will be the value of each part.*

13. When anything is divided into any number of equal parts, each of these parts is called a fractional part of the whole number and is called a *Fraction.*

DEFINITIONS.

1. A FRACTION is one or more of the equal parts of a unit.

2. The unit which is divided is called the UNIT OF THE FRACTION.

3. One of the equal parts into which the unit is divided is a FRACTIONAL UNIT.

Thus, if an apple is divided into four equal parts, the *unit of the fraction* is an *apple,* while the *fractional unit* is a *quarter of an apple.*

4. Two numbers, one written above the other, with a line between them, are used to express a fraction.

Thus, $\frac{2}{3}$ expresses two-thirds, $\frac{7}{8}$ expresses seven-eighths.

5. The DENOMINATOR is the number which shows into how many parts a thing has been divided. It is written below the line.

Thus, in the fraction $\frac{3}{4}$, 4 is the denominator; it shows that a unit has been divided into four equal parts.

6. The NUMERATOR is the number which shows how many parts, expressed by the denominator, are taken. It is written above the line.

Thus, in the fraction $\frac{3}{4}$, 3 is the numerator; it shows that the fraction contains 3 of the four equal parts.

7. A fraction is said to be inverted when its terms are changed one for the other; thus, $\frac{3}{5}$ inverted becomes $\frac{5}{3}$.

8. The Terms of a *Fraction* are the numerator and denominator taken together.

9. A Proper Fraction is one whose numerator is less than its denominator. Its value is less than *one.*

Thus, $\frac{4}{5}$, $\frac{8}{9}$, $\frac{16}{17}$ are proper fractions.

10. An Improper Fraction is one whose numerator equals or exceeds its denominator.

Thus, $\frac{3}{3}$, $\frac{5}{4}$, $\frac{9}{8}$, $\frac{30}{20}$ are improper fractions. The value of an improper fraction is *one* or more than one.

11. A Compound Fraction is a fraction of a fraction, or several fractions with the word *of* between them.

Thus, $\frac{2}{3}$ of $\frac{4}{5}$ and $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{4}{5}$ are compound fractions.

12. A Mixed Number is a number expressed by an integer and a fraction.

Thus, $2\frac{1}{2}$ (read two and one-half), $9\frac{2}{3}$, $18\frac{12}{13}$ are mixed numbers.

13. An Integer is a whole number.

Thus, 2, 5 and 9 are integers.

14. The *Value* of a *Fraction* is the quotient of the numerator divided by the denominator.

Thus, the value of $\frac{3}{4}$ is $3 \div 4$, or $\frac{3}{4}$ of a unit.

15. A fraction expresses *unexecuted* division.

Thus, $\frac{15}{3}$ is equal to $15 \div 3$.

16. Name the numerator and the denominator in each of the following fractions; tell whether the fraction is proper or improper, and why.

1. $\frac{2}{3}$.	*5.* $\frac{5}{6}$.	*9.* $\frac{9}{10}$.	*13.* $\frac{11}{12}$.
2. $\frac{3}{4}$.	*6.* $\frac{7}{7}$.	*10.* $\frac{9}{9}$.	*14.* $\frac{4}{5}$.
3. $\frac{5}{3}$.	*7.* $\frac{8}{7}$.	*11.* $\frac{6}{8}$.	*15.* $\frac{3}{7}$.
4. $\frac{2}{5}$.	*8.* $\frac{4}{3}$.	*12.* $\frac{7}{8}$.	*16.* $\frac{10}{11}$.

17. Give the *unit* of the *fraction*, the *fractional unit*, and the *value* of each of the following:

1. $\frac{2}{3}$ of an apple. *3.* $\frac{2}{9}$. *5.* $\frac{3}{7}$. *7.* $\frac{3}{10}$ of a book.
2. $\frac{3}{5}$ of a dollar. *4.* $\frac{4}{7}$. *6.* $\frac{5}{8}$. *8.* $\frac{5}{6}$ of a lesson.

18. Explain the fraction $\frac{2}{3}$.

EXPLANATION.—In the fraction $\frac{2}{3}$, the denominator, 3, indicates that the unit or thing is divided into 3 equal parts, and the numerator, 2, denotes that 2 of these parts are taken. It may be read 2 of the three equal parts, or one-third of two; it also represents $2 \div 3$; it is read two-thirds.

19. Explain the following fractions:

1. $\frac{3}{4}$. *4.* $\frac{6}{7}$. *7.* $\frac{11}{12}$. *10.* $\frac{13}{32}$.
2. $\frac{4}{5}$. *5.* $\frac{7}{8}$. *8.* $\frac{12}{23}$. *11.* $\frac{42}{59}$.
3. $\frac{5}{9}$. *6.* $\frac{9}{10}$. *9.* $\frac{25}{37}$. *12.* $\frac{51}{63}$.

---

1. When anything is divided into two equal parts, what is each part called? two parts?

2. How many halves in 1? How many thirds in 1?

3. If I have one-half of an apple and you have the rest, what part of it have you? how many halves have we both?

4. If you have $\frac{2}{3}$ of an apple and I have the rest, what part have I? how many thirds have we both?

5. We see, then, that *we may add and subtract fractions, like whole numbers, when they are alike.* They are alike if they are both *halves* or *thirds* or *fourths*, the same as whole numbers are alike when they are both *dollars*, *cents*, etc.

6. How much are two-thirds and one-third? two apples and one apple? $\frac{2}{3}$ and $\frac{1}{3}$?

7. How many are three-fourths and one-fourth? three oranges and one orange? $\frac{3}{4}$ and $\frac{1}{4}$?

1.

REDUCTION OF FRACTIONS.

REDUCTION OF FRACTIONS is the process of changing their form without changing their value.

1. How many halves in 2?

SOLUTION.—Since in 1 there are 2 halves, in 2 there are 2 times 2 halves, or 4 halves (or $\frac{4}{2}$). Therefore, there are 4 halves in 2.

2. How many halves in 4? 5? 6? 8? 10? 12?

3. How many halves in 3 oranges? 5 apples? 9 dollars?

4. How many thirds in 1? 2? 6? 8? 9? 11? 12?

5. How many thirds in $3? $5? $7? $9? $11?

6. How many fourths in 3? 2? 9? 7? 8? 11? 5?

7. How many fifths in 1? 2? 3? 8? 9? 10? 12?

8. How many sixths in 7 cords? 8 dollars? 9 apples?

9. How many sevenths in $3? $5? $8? $11? $12?

10. How many eighths in 2? 3? 4? 5? 6? 7? 8? 9?

11. How many ninths in 1? 6? 7? 8? 9? 10? 11? 12?

12. How many tenths in 2? 4? 6? 7? 12? 9? 8? 5?

13. How many elevenths in 3? 5? 7? 9? 1? 10? 11?

14. How many twelfths in 2? 4? 6? 8? 10? 11? 12?

2.

1. How many whole ones in 4 halves?

SOLUTION.—Since there are 2 halves in 1, in 4 halves there are as many whole ones as 2 is contained times in 4, which is 2 times, or 2 whole ones. Therefore, there are 2 whole ones in 4 halves.

2. How many whole ones in 6 halves? in 8 halves?

3. How many whole ones in 4 halves? in 18 halves?

4. How many whole ones in 10 halves? in 12 halves?

5. How many whole ones in 12 thirds? in 15 thirds?

6. How many whole ones in 8 fourths? in 12 fourths?

7. How many whole ones in 10 fifths? in 15 fifths?

8. How many whole ones in 18 sixths? in 30 sixths?

9. How many whole ones in $\frac{6}{3}$? $\frac{25}{5}$? $\frac{24}{4}$? $\frac{36}{6}$?

10. How many whole ones in $\frac{18}{2}$? $\frac{21}{7}$? $\frac{24}{8}$? $\frac{27}{3}$?

11. How many whole ones in $\frac{36}{4}$? $\frac{33}{11}$? $\frac{144}{12}$? $\frac{81}{9}$?

12. How many whole ones in $\frac{20}{5}$? $\frac{42}{6}$? $\frac{49}{7}$ $\frac{72}{8}$?

3.

1. If you divide 10 apples equally between two boys, what *part* of them will each receive? How *many* will each have?

2. What is $\frac{1}{2}$ of 10?

SOLUTION 1.—$\frac{1}{2}$ of 10 is one of the two equal parts into which 10 can be divided. Therefore, $\frac{1}{2}$ of 10 is 5.

SOLUTION 2.—$\frac{1}{2}$ of 10 is 5, because 2 times 5 are 10. Therefore, $\frac{1}{2}$ of 10 is 5.

3. What is $\frac{1}{3}$ of 15 dollars? What is $\frac{1}{4}$ of 16 bushels?

4. What is $\frac{1}{6}$ of 30 dollars? What is $\frac{1}{6}$ of 42 cents?

5. What is $\frac{1}{7}$ of 28 bushels? What is $\frac{1}{8}$ of 64 dollars?

6. What is $\frac{1}{9}$ of 27 pecks? What is $\frac{1}{10}$ of 40 quarts?

7. What is $\frac{1}{11}$ of 77 apples? What is $\frac{1}{12}$ of 144 children?

8. What is $\frac{1}{9}$ of 63 boys? What is $\frac{1}{7}$ of 49 girls?

9. What is $\frac{1}{11}$ of 121 men? What is $\frac{1}{12}$ of 132 examples?

10. How, then, may we find $\frac{1}{2}$ of a number without the solution?

Answer. Divide the number by 2.

11. How may we find $\frac{1}{3}$ of a number?

12. How may we find $\frac{1}{4}$ of a number?

13. How may we find $\frac{1}{7}$ of a number?

14. How may we find $\frac{1}{8}$ of a number?

15. How may we find $\frac{1}{9}$ of a number?

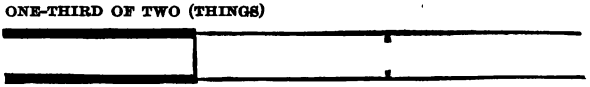
16. How may we find $\frac{1}{11}$ of a number?

17. How may we find $\frac{1}{12}$ of a number?

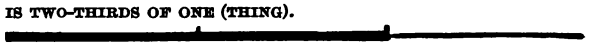
18. A man had 2 dollars and divided it equally among 3 boys; what part of 2 dollars did each get? What part of *one* dollar did each get? Therefore, one-third of 2 dollars is what part of one dollar?

Ans. Two-thirds of 1 dollar.

ONE-THIRD OF TWO (THINGS)



IS TWO-THIRDS OF ONE (THING).



4.

1. What is $\frac{1}{3}$ of 2? What part of 1?

SOLUTION.—$\frac{1}{3}$ of 1 is $\frac{1}{3}$, and $\frac{1}{3}$ of 2 is 2 times $\frac{1}{3}$, which are $\frac{2}{3}$. Therefore, $\frac{1}{3}$ of 2 is $\frac{2}{3}$ of 1.

2. What is $\frac{1}{4}$ of 3? What part of 1?

3. What is $\frac{1}{5}$ of 4? What part of 1?

4. What is $\frac{1}{6}$ of \$5? What part of \$1?

5. How many times is 5 contained in 3?

Ans. $\frac{3}{5}$ times; or $3 \div 5 =$ what? $3 \div 5 = \frac{3}{5}$.

5.

1. A man earned \$15 and spent $\frac{2}{3}$ of it; how many dollars did he spend?

SOLUTION.—He spent $\frac{2}{3}$ of \$15: $\frac{1}{3}$ of \$15 is \$5, and 2 times \$5 are \$10. Therefore, he spent \$10.

2. Mr. Davis traveled 24 miles the first day of his vacation and $\frac{3}{4}$ as far the next day; how far did he travel the second day?

3. Mr. Rogers sold 27 books and Mr. Doty sold $\frac{7}{9}$ as many; how many did Mr. Doty sell?

4. Mr. Gordon spent $60 for a watch and $\frac{2}{3}$ as much for a chain; how much did he spend for the chain?

5. Mr. White is 32 years old and his brother is $\frac{3}{4}$ as old; how old is his brother?

6. If one crate contains 30 boxes of berries, how many boxes in $\frac{2}{3}$ of a crate?

7. I had $64 in the bank; if I drew out $\frac{3}{8}$ of it, how many dollars did I draw out?

8. Find $\frac{3}{4}$ of 28 dollars.

9. Find $\frac{9}{11}$ of 44 dollars.

10. Find $\frac{4}{7}$ of 35 cents.

11. $\frac{4}{9}$ of 36 bushels.

12. $\frac{9}{10}$ of 120 men.

13. $\frac{5}{6}$ of 54 inches.

14. Charles has $\frac{9}{11}$ as many marbles as Frank; if Frank has 33 marbles, how many has Charles?

15. Ned earned $\frac{7}{12}$ as much as George; if George earned $84, how much did Ned earn?

16. Mr. Sage bought a horse for $250; if he bought a carriage for $\frac{3}{5}$ of that sum, how much did he pay for the carriage?

Principles of Fractions.

1. Multiplying the numerator (dividend) of a fraction by any number, multiplies the value of the fraction by that number. Thus, $\frac{1}{8} \times 3 = \frac{3}{8}$.

The number of parts (numerator) is increased, while their size (denominator) remains the same.

2. Dividing the numerator of a fraction by any number, divides the value of the fraction by that number. Thus, $\frac{2}{8} \div 2 = \frac{1}{8}$.

The number of parts is diminished, while their size remains the same.

3. Multiplying the denominator of a fraction by any number, divides the value of the fraction by that number. Thus, $\frac{5}{8} \times 3 = \frac{5}{24}$.

The size of the parts is diminished, while their number remains the same.

4. Dividing the denominator of a fraction by any number, multiplies the fraction by that number. Thus, $\frac{1}{9} \div 3 = \frac{1}{3}$.

The size of the parts is increased, while their number is the same.

5. Multiplying both numerator and denominator of a fraction by the same number, does not change the value of the fraction. Thus, $\frac{1 \times 3}{4 \times 3} = \frac{3}{12}$.

The number of parts is increased as many times as their size is diminished.

6. Dividing both numerator and denominator of a fraction by the same number does not change the value of the fraction. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

The number of parts is diminished as many times as their size is increased.

To Reduce an Improper Fraction to a Whole or Mixed Number.

1. How many units in $\frac{12}{5}$?

Solution.—Since in 1 there are $\frac{5}{5}$, in $\frac{12}{5}$ there are as many units as 5 is contained times in 12, which is 2 and 2 fifths times, or $2\frac{2}{5}$. Therefore, there are $2\frac{2}{5}$ in $\frac{12}{5}$.

2. How many units are there in $\frac{20}{4}$?

3. How many units are there in $\frac{16}{5}$?

4. How many units are there in $\frac{36}{6}$?

5. How many units are there in $\frac{41}{7}$?

6. How many units are there in $\frac{35}{8}$?

7. How many units are there in $\frac{36}{9}$?
8. How many units are there in $\frac{80}{9}$?
9. How many units are there in $\frac{41}{8}$?
10. How many units are there in $\frac{75}{10}$?
11. How many units are there in $\frac{59}{7}$?
12. How many units are there in $\frac{80}{10}$?
13. How many units are there in $\frac{78}{11}$?

How, then, may we reduce an improper fraction to a whole or mixed number?

Ans. Divide the numerator by the denominator and the quotient will be a whole or mixed number.

To Reduce a Mixed Number to an Improper Fraction.

1. Reduce $12\frac{3}{4}$ to fourths.

Solution.—Since in 1 there are 4 fourths, in 12 there are 12 times 4 fourths, or $\frac{48}{4}$, and $\frac{3}{4}$ added equal $\frac{51}{4}$. Therefore, in $12\frac{3}{4}$ there are $\frac{51}{4}$.

2. Reduce $2\frac{1}{2}$ to an improper fraction.
3. Reduce $3\frac{3}{5}$ to an improper fraction.
4. Reduce $4\frac{5}{6}$ to an improper fraction.
5. Reduce $7\frac{4}{5}$ to an improper fraction.
6. Reduce $8\frac{2}{7}$ to an improper fraction.
7. Reduce $9\frac{3}{10}$ to an improper fraction.
8. Reduce $10\frac{2}{3}$ to an improper fraction.
9. Reduce $7\frac{3}{11}$ to an improper fraction.
10. Reduce $12\frac{4}{5}$ to an improper fraction.

How, then, may we reduce a mixed number to an improper fraction?

Ans. Multiply the whole number by the denominator of the fraction; to the product add the numerator and under the sum write the denominator.

A whole number is changed to a fractional form by writing 1 for its denominator.

Thus, $2 = \frac{2}{1}$. The denominator, 1, changes the form of the whole number without altering its value.

1. Change 5 to the form of a fraction.

2. Change 7 to the form of a fraction.

3. Change 10 to the form of a fraction.

To Reduce a Fraction to a Higher Denomination.

1. Change $\frac{3}{4}$ to twelfths.

Solution.—Since in 1 there are $\frac{12}{12}$, in $\frac{1}{4}$ of 1 there are $\frac{1}{4}$ of $\frac{12}{12}$, which are $\frac{3}{12}$; and in $\frac{3}{4}$ there are 3 times $\frac{3}{12}$, which are $\frac{9}{12}$. Therefore, in $\frac{3}{4}$ there are $\frac{9}{12}$.

2. Change $\frac{1}{2}$ and $\frac{2}{3}$ to 6ths; $\frac{2}{3}$ and $\frac{4}{5}$ to 15ths.

3. Change $\frac{2}{3}$ and $\frac{3}{5}$ to 30ths; $\frac{3}{4}$ and $\frac{7}{8}$ to 16ths.

4. Change $\frac{4}{5}$ and $\frac{5}{7}$ to 70ths; $\frac{5}{6}$ and $\frac{7}{9}$ to 18ths.

5. Reduce $\frac{2}{3}$, $\frac{3}{4}$ and $\frac{5}{6}$ to twelfths.

6. Reduce $\frac{1}{2}$, $\frac{3}{4}$ and $\frac{5}{8}$ to sixteenths.

How, then, may we reduce a fraction to a required denominator?

Ans. Multiply both numerator and denominator by the number which will give the required denominator.

To Reduce a Fraction from Higher to Lower Terms.

1. Change $\frac{8}{12}$ to thirds.

Solution.—One equals $\frac{12}{12}$ and $\frac{1}{3}$ equals $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$. Since $\frac{4}{12}$ equals $\frac{1}{3}$, $\frac{8}{12}$ equals as many thirds as $\frac{4}{12}$ is contained times in $\frac{8}{12}$, which is 2 times, or $\frac{2}{3}$. Therefore, $\frac{8}{12}$ equals $\frac{2}{3}$.

2. Change $\frac{3}{6}$ to halves.

3. Change $\frac{8}{12}$ to thirds.

4. $\frac{10}{20}$ to fourths.

5. $\frac{6}{9}$ to thirds.

6. Change $\frac{10}{25}$ to fifths.
7. Change $\frac{16}{20}$ to fifths.
8. Change $\frac{9}{12}$ to fourths.
9. Change $\frac{12}{18}$ to sixths.
10. Change $\frac{12}{21}$ to sevenths.
11. $\frac{12}{24}$ to sixths.
12. $\frac{14}{49}$ to sevenths.
13. $\frac{6}{24}$ to eighths.
14. $\frac{18}{48}$ to eighths.
15. $\frac{28}{36}$ to ninths.

How, then, may we reduce a fraction from higher to lower terms?

Ans. Divide both numerator and denominator by the number which will give the required denominator.

To Reduce a Compound Fraction to a Simple Fraction.

1. What is $\frac{1}{2}$ of $\frac{1}{4}$?

Solution.—$\frac{1}{2}$ of $\frac{1}{4}$ is one of the two equal parts into which $\frac{1}{4}$ may be divided; if each $\frac{1}{4}$ is divided into 2 equal parts, 4 fourths, or 1, will be divided into 4 times 2, or 8, equal parts; hence, each part is $\frac{1}{8}$ of 1, or $\frac{1}{8}$. Therefore, $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$.

Note.—The word *of* between the fractions is equivalent to the *sign of multiplication.*

In reducing a compound fraction to a simple one, what do we do with the numerators and what with the denominators?

2. What is $\frac{1}{2}$ of $\frac{1}{3}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{7}$? $\frac{2}{3}$ of $\frac{1}{4}$?

3. What is $\frac{1}{4}$ of $\frac{1}{7}$? $\frac{1}{5}$ of $\frac{1}{6}$? $\frac{1}{5}$ of $\frac{1}{10}$? $\frac{1}{12}$ of $\frac{1}{10}$? $\frac{3}{4}$ of $\frac{2}{5}$?

$\frac{3}{4} \times \frac{2}{5} = \frac{6}{20} = \frac{3}{10}$, or,

Cancelling or rejecting equal factors from both numerator and denominator.

Thus, $\frac{3}{\cancel{4}_{2}} \div \frac{\cancel{2}}{5} = \frac{3}{10}$.

How, then, may we reduce a compound fraction to a simple one?

Ans. Multiply the numerators together and the denominators together.

To Reduce a Fraction to Its Lowest Terms.

A fraction is in its lowest terms when no number greater than 1 will exactly divide both numerator and denominator.

1. Reduce $\frac{9}{12}$ to its lowest terms.

Solution. — Dividing both terms of a fraction by the same number does not alter the value of the fraction (Prin. 6); hence, we divide both numerator and denominator by 3 and obtain $\frac{3}{4}$. As these terms, 3 and 4, have no number which will exactly divide each of them, the fraction is in its lowest terms. Therefore, $\frac{9}{12}=\frac{3}{4}$.

2. Reduce $\frac{2}{8}$ to its lowest terms.
3. Reduce $\frac{4}{6}$ to its lowest terms.
4. Reduce $\frac{3}{9}$ to its lowest terms.
5. Reduce $\frac{5}{10}$ to its lowest terms.
6. Reduce $\frac{12}{18}$ to its lowest terms.
7. Reduce $\frac{4}{12}$ to its lowest terms.
8. Reduce $\frac{8}{24}$ to its lowest terms.
9. Reduce $\frac{9}{27}$ to its lowest terms.
10. Reduce $\frac{10}{30}$ to its lowest terms.
11. Reduce $\frac{12}{21}$ to its lowest terms.
12. Reduce $\frac{63}{81}$ to its lowest terms.
13. Reduce $\frac{9}{54}$ to its lowest terms.
14. Reduce $\frac{16}{24}$ to its lowest terms.
15. Reduce $\frac{21}{28}$ to its lowest terms.
16. Reduce $\frac{20}{30}$ to its lowest terms.
17. Reduce $\frac{35}{45}$ to its lowest terms.
18. Reduce $\frac{77}{99}$ to its lowest terms.
19. Reduce $\frac{56}{64}$ to its lowest terms.

How, then, may we reduce a fraction to its lowest terms?

Ans. Divide both numerator and denominator by the factors common to both.

COMMON DENOMINATOR.

1. A COMMON DENOMINATOR is a denominator common to several fractions, or a denominator to which all may be reduced. It is a denominator exactly divisible by the several denominators of the given fractions.

2. SIMILAR FRACTIONS are those that have the same denominator.

Thus, $\frac{2}{3}$ and $\frac{1}{3}$ are similar fractions.

3. DISSIMILAR FRACTIONS are those that have different denominators.

Thus, $\frac{3}{5}$ and $\frac{4}{9}$ are dissimilar fractions.

TO REDUCE FRACTIONS TO A COMMON DENOMINATOR.

1. Reduce $\frac{2}{3}$ and $\frac{3}{4}$ to a common denominator.

SOLUTION.—A common denominator for *thirds* and *fourths* is *twelfths*. In 1 there are $\frac{12}{12}$, and in $\frac{1}{3}$ there are $\frac{1}{3}$ of $\frac{12}{12}$, or $\frac{4}{12}$; in $\frac{2}{3}$ there are 2 times $\frac{4}{12}$, or $\frac{8}{12}$. In $\frac{3}{4}$ there are 3 times $\frac{1}{4}$ of $\frac{12}{12}$; $\frac{1}{4}$ of $\frac{12}{12}$ is $\frac{3}{12}$, and 3 times $\frac{3}{12}$ are $\frac{9}{12}$. Therefore, $\frac{2}{3}$ and $\frac{3}{4}$ are equivalent to $\frac{8}{12}$ and $\frac{9}{12}$.

2. Reduce $\frac{1}{4}$ and $\frac{1}{5}$ to equivalent fractions having a common denominator.

3. Reduce $\frac{2}{3}$ and $\frac{3}{5}$ to equivalent fractions having a common denominator.

4. Reduce $\frac{3}{4}$ and $\frac{4}{5}$ to equivalent fractions having a common denominator.

5. Reduce $\frac{5}{6}$ and $\frac{6}{7}$ to equivalent fractions having a common denominator.

6. Reduce $\frac{6}{7}$ and $\frac{7}{8}$ to equivalent fractions having a common denominator.

7. Reduce $\frac{7}{8}$ and $\frac{8}{9}$ to equivalent fractions having a common denominator.

8. Reduce $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{5}$ to equivalent fractions having a common denominator.

9. Reduce $\frac{1}{7}$, $\frac{1}{9}$ and $\frac{2}{3}$ to equivalent fractions having a common denominator.

10. Reduce $\frac{1}{9}$ and $\frac{1}{11}$ to equivalent fractions having a common denominator.

11. Reduce $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{4}{7}$ to equivalent fractions having a common denominator.

12. Reduce $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$ to equivalent fractions having a common denominator.

13. Reduce $\frac{1}{2}$, $\frac{3}{5}$ and $\frac{1}{6}$ to equivalent fractions having a common denominator.

14. Reduce $\frac{9}{11}$ and $\frac{11}{12}$ to equivalent fractions having a common denominator.

ADDITION OF FRACTIONS.

To add two or more fractions, they must be similar.

1. John gave \$$\frac{1}{2}$ for a book and \$$\frac{1}{3}$ for a knife; what part of a dollar did he spend?

SOLUTION.—He spent the sum of \$$\frac{1}{2}$ and \$$\frac{1}{3}$. As the fractions are dissimilar, we reduce them to a common denominator. A common denominator for *halves* and *thirds* is *sixths;* $\frac{1}{2}$ is equal to $\frac{3}{6}$, and $\frac{1}{3}$ is equal to $\frac{2}{6}$; $\frac{3}{6}$ and $\frac{2}{6}$ are $\frac{5}{6}$. Therefore, he spent \$$\frac{5}{6}$.

2. A clerk spent $\frac{1}{5}$ of his salary for board and $\frac{1}{10}$ of it for other expenses; what part of his salary did he spend?

3. A man owned $\frac{2}{5}$ of a vessel and his wife $\frac{3}{8}$ of it; what part of the vessel did both own?

4. A lady traveled $\frac{1}{3}$ of her entire journey the first day and $\frac{1}{8}$ of it the second day; what part did she travel during both days?

5. I paid \$$\frac{1}{2}$ for turnips and \$$\frac{2}{3}$ for potatoes; how much did I spend?

6. A boy worked $\frac{2}{5}$ of a day for one man and $\frac{1}{4}$ of a day for another; what part of a day did he work for both?

7. James spent $\frac{2}{9}$ of a dollar for a kite and $\frac{1}{6}$ of a dollar for a collar; what part of a dollar did he give for both?

8. A farmer sold $\frac{1}{4}$ of his sheep to one man and $\frac{1}{2}$ to another; what part of his flock did he sell to both?

9. A boy had $\frac{4}{5}$ of a dollar and his brother gave him $\frac{1}{10}$ of a dollar; what part of a dollar had he then?

10. Hezekiah spent $\frac{1}{4}$ of a dollar for a cap and $\frac{1}{2}$ of a dollar for a book; what part of a dollar did he spend?

11. A man sold $\frac{3}{5}$ of an acre to one man and $\frac{2}{3}$ of an acre to another; how much did he sell to both?

12. A farmer sold $\frac{1}{4}$ of his grain to one man, $\frac{1}{5}$ to another and $\frac{1}{10}$ to another; how much did he sell?

13. Martha worked $\frac{1}{3}$ of her examples in the evening and $\frac{5}{6}$ of them the next morning; what part of them had she then finished?

14. Matilda spent $\frac{1}{4}$ of her money for a watch and $\frac{1}{5}$ of it for a chain; what part of her money did she spend?

15. Mr. Livingston spent $\frac{1}{5}$ of his money to reach Chicago, and $\frac{4}{5}$ of it while there; what part of his money did he spend?

16. Mr. Brewster gave $\frac{1}{12}$ of his money to one hospital and $\frac{1}{5}$ of it to another; what part of his money did he give to both?

17. Johnnie spent $\frac{1}{10}$ of his money for ice cream, $\frac{1}{5}$ of it for soda water, and $\frac{1}{12}$ of it for peanuts; what part of the whole did he spend?

18. Jane spent $\frac{1}{3}$ of her money for breakfast, $\frac{1}{2}$ of it for dinner, and $\frac{1}{10}$ of it for supper; what part of her money did she spend?

19. Mary spent $\frac{2}{3}$ of her time at Philadelphia and $\frac{1}{7}$ of it at Bangor; what part of her time did she spend at both places?

20. Fred received $\frac{1}{5}$ of a dollar from his mother and $\frac{1}{2}$ of a dollar from his father; what part of a dollar did he receive from both?

21. George spent $\frac{2}{5}$ of a dollar for a book, $\frac{1}{4}$ of a dollar for some paper, and $\frac{3}{10}$ of a dollar for some pencils; how much did he spend?

22. Mr. Emerson spent $\frac{3}{10}$ of his money for ice and $\frac{5}{9}$ of it for coal; what part of his money did he expend?

Find the sum of

23. One-third and one-fourth.

24. Two-thirds and three-fourths.

SOLUTION.—$\frac{2}{3}$ equals $\frac{8}{12}$, and $\frac{3}{4}$ equals $\frac{9}{12}$; $\frac{8}{12}$ and $\frac{9}{12}$ are $\frac{17}{12}$, or $1\frac{5}{12}$. Therefore, the sum of $\frac{2}{3}$ and $\frac{3}{4}$ is $1\frac{5}{12}$.

25. Three-fourths and four-fifths.

26. One-half and two-thirds.

27. Three-fifths and four-sevenths.

28. What is the sum of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$?

29. What is the sum of $\frac{4}{5}$ and $\frac{1}{3}$?

30. What is the sum of $\frac{1}{3}$, $\frac{1}{8}$ and $\frac{1}{2}$?

31. What is the sum of $1\frac{1}{2}$ and $1\frac{1}{3}$?

NOTE.—Add the whole numbers and the fractions separately and unite their sum. Reduce fractional results to lowest terms.

Thus, in $1\frac{1}{3}$ plus $1\frac{1}{6}$. The sum of $\frac{1}{3}$ and $\frac{1}{6}$ is $\frac{5}{6}$; the sum of 1 and 1 is 2, and $\frac{5}{6}$ added makes $2\frac{5}{6}$, the sum.

32. What is the sum of $1\frac{2}{3}$ and $4\frac{5}{6}$?

33. What is the sum of $1\frac{1}{2}$, $2\frac{1}{3}$ and $3\frac{1}{4}$?

34. What is the sum of $\frac{3}{4}$, $\frac{1}{5}$ and $\frac{3}{10}$?

35. What is the sum of $1\frac{3}{4}$, $3\frac{5}{8}$ and $3\frac{1}{16}$?

SUBTRACTION OF FRACTIONS.

To subtract fractions, they must be similar.

1. A boy sold a book for $\$\frac{3}{4}$ that cost him $\$\frac{2}{5}$; what part of a dollar did he gain?

SOLUTION. — He gained the difference between $\$\frac{3}{4}$ and $\$\frac{2}{5}$. Since the fractions are dissimilar, they must be reduced to a common denominator. A common denominator for *fourths* and *fifths* is *twentieths*. $\$\frac{3}{4}$ equals $\$\frac{15}{20}$, and $\$\frac{2}{5}$ equals $\$\frac{8}{20}$; the difference between $\$\frac{15}{20}$ and $\$\frac{8}{20}$ is $\$\frac{7}{20}$. Therefore, he gained $\$\frac{7}{20}$.

2. I paid $\$2\frac{1}{2}$ for a book and sold it for $\$4\frac{1}{6}$; how much did I gain?

SOLUTION.—I gained the difference between $\$4\frac{1}{6}$ and $\$2\frac{1}{2}$; $\$2\frac{1}{2}$ equals $\$2\frac{3}{6}$; we cannot take $\$\frac{3}{6}$ from $\$\frac{1}{6}$, so we take $\$1$, or $\$\frac{6}{6}$ (from $\$4$), which, added to $\$\frac{1}{6}$, becomes $\$\frac{7}{6}$. $\$\frac{3}{6}$ from $\$\frac{7}{6}$ leaves $\$\frac{4}{6}$, or $\$\frac{2}{3}$, and $\$2$ from $\$3$ leaves $\$1$. Uniting the remainders, $\$1$ and $\$\frac{2}{3}$, we have $\$1\frac{2}{3}$. Therefore, I gained $\$1\frac{2}{3}$ by the sale.

Process: $\$4\frac{1}{6} = \$3\frac{7}{6}$

$$\begin{array}{r} 2\frac{1}{2} = \ \ 2\frac{3}{6} \\ \hline \$1\frac{4}{6} = \$1\frac{2}{3}. \end{array}$$

NOTE.—Subtract mixed numbers by subtracting integers and fractions separately, and unite the remainders.

3. George had $\frac{9}{10}$ of a dollar and spent $\frac{1}{2}$ of a dollar; what part of a dollar had he left?

4. Mr. Weld bought $3\frac{1}{5}$ acres of land and sold $2\frac{1}{10}$ acres; how much land has he left?

5. A man hoed $\frac{5}{6}$ of a field of corn in one day; if he hoed $\frac{1}{3}$ in the forenoon, what part did he hoe in the afternoon?

6. Judson earned $\frac{5}{8}$ of a dollar and Charles earned $1\frac{1}{4}$ dollars; how much more did Charles earn than Judson?

7. If Louise bought a toy for $\$\frac{3}{5}$ and gave the merchant $\$1$, what part of a dollar should she receive as her change? ($\$1$ equals $\$\frac{5}{5}$.)

8. Frank had $\$1\frac{1}{2}$ and spent $\$\frac{3}{4}$; how much had he left?

9. Anna had $\frac{4}{5}$ of a dollar and spent $\frac{1}{4}$ of a dollar; what part of a dollar has she left?

10. What is the difference between $3\frac{1}{8}$ pecks and $2\frac{1}{3}$ pecks?

11. Samuel and Clyde picked $3\frac{1}{2}$ bushels of beans; if Samuel picked $1\frac{8}{9}$ bushels, how many bushels did Clyde pick?

12. I gave $\$3\frac{1}{2}$ for a pair of shoes and $\$7\frac{1}{4}$ for a hat; how much more did I give for the hat than for the shoes?

13. After spending a part of my money, I had $\frac{3}{20}$ of it left; what part did I spend?

NOTE.—The *whole* is represented by $\frac{20}{20}$.

14. Mrs. Ford received $\frac{2}{5}$ of her father's fortune and Mrs. Nellis received $\frac{3}{7}$ of it; how much greater was the share of Mrs. Nellis than that of Mrs. Ford?

15. Mr. Swift traveled $3\frac{8}{9}$ miles in one hour and Mr. Woodbury $2\frac{7}{8}$ miles; how much farther did one travel in an hour than the other?

16. From a field containing $4\frac{1}{6}$ acres, $2\frac{1}{3}$ acres were sold; how many remained unsold?

17. How many are $3\frac{1}{2}$ pints less $2\frac{1}{4}$ pints?

18. Albert bought $5\frac{1}{6}$ bushels of potatoes yesterday and sold $2\frac{3}{4}$ bushels to-day; how many bushels had he left?

19. Mr. Goldstein bought an umbrella for $\$6\frac{3}{4}$ and a hat for $\$4\frac{4}{5}$; how much more did he pay for the umbrella than for the hat?

20. It took me $\frac{1}{60}$ of an hour to make a mistake and $1\frac{1}{4}$ hours to rectify it; how much longer did it take me to rectify my error than to make it?

21. The minuend is $3\frac{1}{11}$; the subtrahend $\frac{6}{7}$; find the remainder.

What is the difference between—

22. $\frac{3}{4}$ and $\frac{2}{3}$? \$$\frac{1}{2}$ and \$$\frac{1}{4}$?

SOLUTION 1.—$\frac{3}{4}$ equals $\frac{9}{12}$, and $\frac{2}{3}$ equals $\frac{8}{12}$; the difference between $\frac{9}{12}$ and $\frac{8}{12}$ is $\frac{1}{12}$. Therefore, the difference between $\frac{3}{4}$ and $\frac{2}{3}$ is $\frac{1}{12}$.

SOLUTION 2.—\$$\frac{1}{2}$ equals \$$\frac{2}{4}$; the difference between \$$\frac{3}{4}$ and \$$\frac{1}{4}$ is \$$\frac{1}{4}$. Therefore, the difference between \$$\frac{1}{2}$ and \$$\frac{1}{4}$ is \$$\frac{1}{4}$.

23. $\frac{1}{3}$ and $\frac{1}{7}$? $\frac{1}{9}$ of a bushel and $\frac{1}{11}$ of a bushel?

24. $\frac{2}{3}$ and $\frac{3}{4}$? $\frac{3}{4}$ of a pint and $\frac{2}{3}$ of a pint?

25. $\frac{3}{5}$ and $\frac{3}{6}$? $\frac{1}{3}$ of a quart and $\frac{1}{12}$ of a quart?

26. $\frac{3}{8}$ and $\frac{3}{10}$? $\frac{7}{8}$ of a dollar and $\frac{4}{5}$ of a dollar?

27. From $4\frac{2}{3}$ take $2\frac{1}{3}$. Take $3\frac{1}{2}$ from 12.

28. From $5\frac{1}{8}$ take $2\frac{1}{2}$. Take $3\frac{2}{5}$ from $5\frac{1}{4}$.

29. From $6\frac{1}{2}$ take $4\frac{3}{4}$. Take $\frac{17}{18}$ from $1\frac{5}{6}$.

30. What is the difference between $3\frac{1}{4}$ and $1\frac{2}{3}$?

31. What is the difference between $4\frac{1}{8}$ and $2\frac{1}{3}$?

32. What is the difference between 33 and $5\frac{1}{6}$?

33. What is the difference between $14\frac{1}{9}$ and $8\frac{1}{3}$?

34. What is the difference between $2\frac{1}{7}$ and $1\frac{4}{9}$?

PROBLEMS COMBINING ADDITION AND SUBTRACTION.

1. After giving $\frac{1}{3}$ of my money to one son and $\frac{4}{9}$ of it to another, how much have I left?

SOLUTION.—I have left the difference between $\frac{9}{9}$, or the whole, and the sum of $\frac{1}{3}$ and $\frac{4}{9}$. $\frac{1}{3}$ equals $\frac{3}{9}$; $\frac{3}{9}$ and $\frac{4}{9}$ are $\frac{7}{9}$, and $\frac{9}{9}$ minus $\frac{7}{9}$ equals $\frac{2}{9}$. Therefore, I have $\frac{2}{9}$ of my money left.

2. Mr. Page sold $\frac{1}{5}$ of his farm to one man, $\frac{1}{3}$ to another, and $\frac{1}{4}$ to another; what part of his farm had he left?

3. Mrs. Arnold paid $\frac{1}{4}$ of her money for a hat, $\frac{1}{5}$ of it for a pair of shoes, and $\frac{1}{10}$ of it for a pair of gloves; what part of her money has she left?

4. Isaac divided $\$21\frac{1}{2}$ among three persons, giving to the first $\$5\frac{1}{4}$, to the second $\$10\frac{1}{2}$, and to the third the remainder; how much did he give to the third?

5. The sum of three numbers is $15\frac{1}{4}$; two of them are $4\frac{1}{8}$ and $4\frac{1}{2}$; what is the third?

6. Frank is 9 years old and George is $4\frac{1}{8}$ years less than twice as old as Frank; what is the sum of their ages?

7. From a piece of cloth containing 30 yards, $4\frac{1}{6}$ yards were cut at one time, $5\frac{1}{4}$ yards at another, and $2\frac{1}{6}$ at another; how many yards remained in the piece?

8. I deposited in the bank at one time $\$10\frac{1}{2}$, at another $\$20\frac{1}{10}$; how much more must I deposit to make $50 in all?

9. Charles Augustus had $10; he spent $\$3\frac{1}{2}$ for a cane and $\$2\frac{1}{4}$ for a hat; how much had he left?

10. Mr. Young earned $\$2\frac{1}{4}$ a day, but was obliged to spend $\$\frac{7}{8}$ a day for board and $\$\frac{3}{4}$ for other expenses; how much did he save in one day?

11. Mr. Otis spent $\frac{1}{2}$ of his money for board, $\frac{1}{6}$ of it for rent, $\frac{1}{8}$ of it for clothing, and $\frac{1}{12}$ of it for books; what part of his money had he left?

12. Mr. Little sold $2\frac{1}{2}$ bushels of potatoes to one man, $2\frac{1}{4}$ bushels to another, and $3\frac{3}{4}$ bushels to another; if he had at first 12 bushels, how many has he left?

13. A boy paid $\frac{3}{5}$ of a dollar for an arithmetic, $\frac{3}{4}$ of a dollar for a grammar, and $\frac{4}{5}$ of a dollar for an algebra; how much change ought he to receive from a 5-dollar bill?

14. Fred spent $\frac{1}{2}$ of a dollar for soap, $\frac{1}{8}$ of a dollar for sugar, and $\frac{1}{4}$ of a dollar for fruit, on Monday; on Tues-

day he spent \$$1\frac{1}{3}$ for meat, \$$1\frac{1}{4}$ for onions, and \$$1\frac{1}{5}$ for corn; how much more did he spend on Monday than on Tuesday?

15. Ross spent $\frac{1}{2}$ of his money for a dinner, $\frac{1}{4}$ of it for a book, and $\frac{1}{12}$ of it for a pencil; what part of his money has he left?

16. Irene is $14\frac{1}{3}$ years old; how old was she $2\frac{5}{9}$ years ago?

17. John Henry is $8\frac{5}{6}$ years old; how old was he $3\frac{3}{4}$ years ago?

18. Ned earned \$$2\frac{1}{7}$ on Monday and \$$4\frac{1}{4}$ on Tuesday; Wednesday he spent \$$1\frac{1}{2}$ and Thursday \$$1\frac{3}{4}$; how much did he have left Thursday night?

19. Three boys together gathered $4\frac{1}{2}$ quarts of nuts; one gathered $1\frac{1}{3}$ quarts, another $2\frac{1}{4}$ quarts; how many did the third boy gather?

20. A lady bought the material for a dress for \$$16\frac{1}{2}$ and paid \$$7\frac{1}{4}$ for having it made; how much less than \$25 did it cost her?

21. The larger of two numbers is $10\frac{1}{4}$ and the smaller is $3\frac{1}{2}$ less than the larger; what is the sum of the numbers?

22. What is the difference between 14, and $2\frac{1}{3}$ plus $8\frac{1}{9}$?

23. From the sum of $1\frac{5}{6}$ and $8\frac{2}{3}$ take $4\frac{1}{2}$.

24. Take the sum of $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ from $2\frac{1}{3}$.

25. Take the sum of $2\frac{1}{2}$ and $4\frac{1}{8}$ from the sum of $3\frac{1}{3}$ and $5\frac{1}{2}$.

26. From $40 - 12\frac{1}{2}$ take $2\frac{1}{3}$ plus $5\frac{1}{9}$.

27. From $2 + \frac{1}{4} + \frac{1}{5}$ take $1\frac{13}{20}$.

28. Take the sum of $5\frac{1}{8}$ and $2\frac{1}{4}$ from the difference between 40 and $22\frac{1}{3}$.

29. Take $1\frac{1}{2} + 2\frac{1}{3} + 3\frac{1}{4}$ from $14\frac{1}{12}$.

30. What kind of fractions only can be added and subtracted?

31. What must be done with dissimilar fractions before they can be added or subtracted?

32. What is meant by common denominator?

MULTIPLICATION OF FRACTIONS.

To Multiply a Fraction by an Integer.

1. If a yard of cloth cost $\$\frac{8}{5}$, how much will 4 yards cost?

Solution.—Since 1 yard cost $\$\frac{8}{5}$, 4 yards will cost 4 times $\$\frac{8}{5}$, or $\$\frac{32}{5}$, equal to $\$1\frac{2}{5}$. Therefore, 4 yards will cost $\$1\frac{2}{5}$.

A fraction is multiplied by multiplying its numerator (Prin. 1). We divide the denominator when it will contain the multiplier an exact number of times (Prin. 4).

2. What will 4 yards of cloth cost at $\$1\frac{1}{8}$ a yard?

Solution.—Since 1 yard cost $\$1\frac{1}{8}$, 4 yards will cost 4 times $\$1\frac{1}{8}$ (or $\$\frac{9}{8}$), which are $\$\frac{9}{2} = \$4\frac{1}{2}$. Therefore, 4 yards will cost $\$4\frac{1}{2}$.

Note.—Reduce mixed numbers to improper fractions and multiply numerator or divide denominator.

3. What will 20 pounds of butter cost at $\$\frac{1}{5}$ a pound?

4. If a box of raisins cost $\$2\frac{1}{2}$, what will 8 boxes cost?

5. If a man can reap $\frac{4}{7}$ of an acre of rye in a day, how much can 14 men reap?

6. How much money was given to 16 poor families in one day, if each family received $\$\frac{3}{4}$?

7. What will 10 tickets cost at $\$\frac{2}{5}$ apiece?

8. At $\$\frac{7}{8}$ a yard, what will 24 yards of silk cost?

9. How much will 11 books cost at $\$\frac{4}{5}$ each?

10. If a man earn $3\frac{1}{8}$ dollars a day, how much can he earn in 3 days?

11. If sugar is worth $6\frac{1}{4}¢$ a pound, what are 5 pounds worth?

12. What will 4 pounds of rice cost at $7\frac{1}{2}¢$ a pound?

13. What will 10 hats cost at $\$4\frac{1}{2}$ apiece?

14. What will 12 dozen buttons cost at $\$\frac{3}{8}$ a dozen?

15. If one dinner cost $\$\frac{3}{4}$, what will 9 dinners cost?

16. What will 13 pounds of flour cost at $4\frac{1}{2}$ cents a pound?

17. If one fruit-jar hold $\frac{3}{8}$ of a gallon, what will 7 jars hold?

To Multiply an Integer by a Fraction.

1. At \$15 a ton, what will $\frac{2}{3}$ of a ton of hay cost?

Solution.—Since 1 ton cost \$15, $\frac{2}{3}$ of a ton will cost $\frac{2}{3}$ of \$15. $\frac{1}{3}$ of \$15 is \$5, and 2 times \$5 are \$10. Therefore, $\frac{2}{3}$ of a ton will cost \$10, at \$15 a ton.

2. If a load of wheat cost \$50, what will $\frac{3}{4}$ of it cost?

Solution.—Since one load cost \$50, $\frac{3}{4}$ of a load will cost 3 times $\frac{1}{4}$ of \$50. $\frac{1}{4}$ of \$50 is $\$\frac{50}{4}$, and 3 times $\$\frac{50}{4}$ are $\$\frac{150}{4} = \$37\frac{1}{2}$. Therefore, $\frac{3}{4}$ of a load will cost $\$37\frac{1}{2}$, at \$50 a load.

3. What will $5\frac{1}{2}$ pounds of meat cost at $12¢$ a pound?

4. At $15¢$ a peck, what will $3\frac{1}{2}$ pecks of potatoes cost?

5. What will $3\frac{3}{4}$ dozen bananas cost at $16¢$ a dozen?

Note.—Pupils should learn to multiply by the following

Process—multiplication.

	$16¢$ multiplied by $\frac{3}{4} = 12¢$	
	$16¢$ " " $3 = 48¢$	
$3\frac{3}{4}$ times $16¢ =$		$60¢$

6. At \$25 an acre, what will $4\frac{1}{2}$ acres of land cost?

7. What will $5\frac{5}{6}$ pounds of cheese cost at 10 cents a pound?

8. If a man travel 6 miles an hour, how far will he travel in $7\frac{1}{2}$ hours?

9. If I read 6 books in one week, how many books can I read in $5\frac{1}{7}$ weeks?

10. If car fare is 3 cents a mile, what will it cost to ride $42\frac{1}{6}$ miles?

11. At \$5 a ton, what will $6\frac{2}{3}$ tons of coal cost?

12. What will $5\frac{7}{8}$ yards of cloth cost at 9¢ a yard?

13. At 12¢ a gallon, what will $5\frac{3}{4}$ gallons of gasoline cost?

14. If Mary can hem 12 towels in one day, how many can she hem in $5\frac{1}{2}$ days?

15. At \$6 a week, what will be the rent of a house for $7\frac{1}{2}$ weeks?

16. What will $21\frac{1}{2}$ pounds of iron cost at 5¢ a pound?

17. If one bag will hold 2 bushels of grain, how many bushels will $15\frac{1}{3}$ bags hold?

18. At \$15 a barrel, what will $6\frac{2}{3}$ barrels of fish cost?

19. At 2 dollars a yard, what will $15\frac{3}{4}$ yards of silk cost?

20. At 12¢ a pound, what will $12\frac{1}{2}$ pounds of meat cost?

To Multiply a Fraction by a Fraction.

1. A man having $\$\frac{2}{3}$ gave away $\frac{3}{4}$ of it; how much did he give away?

Solution.—If he gave away $\frac{3}{4}$ of it, he gave away 3 times $\frac{1}{4}$ of $\$\frac{2}{3}$. $\frac{1}{4}$ of $\$\frac{2}{3}$ is $\$\frac{2}{12}$, and 3 times $\$\frac{2}{12}$ are $\$\frac{6}{12}$, or $\$\frac{1}{2}$. Therefore, he gave away $\$\frac{1}{2}$.

2. A man owning $\frac{3}{5}$ of an iron foundry sold $\frac{1}{3}$ of his share; what part of the foundry did he sell?

3. If I own $\frac{2}{3}$ of an acre of land and sell $\frac{1}{7}$ of it, what part of an acre do I sell?

4. Sarah is $\frac{1}{3}$ as old as her mother and Ruth is $\frac{1}{6}$ as old as Sarah; what part of her mother's age is Ruth's age?

5. Arthur, having $\frac{4}{5}$ of a dollar, gave away $\frac{2}{3}$ of it; what part of a dollar did he give away?

6. Grace earned $\frac{7}{8}$ of a dollar and on her way home spent $\frac{1}{3}$ of it; what part of a dollar did she spend?

7. What will be the cost of $\frac{5}{8}$ of a bushel of beans at $\$\frac{8}{11}$ a bushel?

8. At $\$\frac{3}{5}$ a pound, what will $5\frac{1}{3}$ pounds of tea cost?

SOLUTION 1.—Since one pound cost $\$\frac{3}{5}$, $5\frac{1}{3}$ pounds will cost $5\frac{1}{3}$ times $\$\frac{3}{5}$. 5 times $\$\frac{3}{5}$ are $\$\frac{15}{5}$, or $\$3$, and $\frac{1}{3}$ of $\$\frac{3}{5}$ is $\$\frac{1}{5}$, which, added to $\$3$, makes $\$3\frac{1}{5}$. Therefore, $5\frac{1}{3}$ pounds will cost $\$3\frac{1}{5}$.

SOLUTION 2.—Since one pound cost $\$\frac{3}{5}$, $5\frac{1}{3}$ pounds will cost $5\frac{1}{3}$, or $\frac{16}{3}$ times $\$\frac{3}{5}$, or $\$\frac{48}{15}$, which equals $\$3\frac{3}{15}$, or $\$3\frac{1}{5}$. Therefore, etc.

9. At $\$6\frac{1}{4}$ a bushel, what will $\frac{2}{5}$ of a bushel of clover seed cost?

10. At $\$3\frac{1}{2}$ a day, how much can a man earn in $\frac{2}{3}$ of a day?

11. What will $5\frac{1}{6}$ yards of ribbon cost at $\$\frac{3}{5}$ a yard?

12. At $\$\frac{2}{3}$ a bushel, what will $8\frac{1}{4}$ bushels of peaches cost?

13. At $\$\frac{3}{4}$ a rod, what will it cost to dig a ditch $\frac{1}{2}$ of $\frac{1}{3}$ of 26 rods long?

14. If a man walks $3\frac{2}{5}$ miles in an hour, how far can he walk in $\frac{5}{6}$ hours?

15. If a yard of cloth cost $\$\frac{2}{3}$, what part of a dollar will $\frac{5}{9}$ of a yard cost?

16. What part of his farm does a man sell who sells $\frac{3}{4}$ of $\frac{8}{9}$ of it?

17. At $\$3\frac{1}{2}$ a yard, what will $4\frac{1}{4}$ yards of broadcloth cost?

NOTE.—Reduce mixed numbers to improper fractions.

18. A man having $\$\frac{4}{5}$ gave $\frac{2}{3}$ of it for a knife; how much had he left?

SOLUTION.—Since he gave $\frac{2}{3}$ of it for a knife, he must have had left the difference between $\frac{3}{3}$ and $\frac{2}{3}$, or $\frac{1}{3}$ of it; or $\frac{1}{3}$ of $\$\frac{4}{5}$, which equals $\$\frac{4}{15}$. Therefore, he had $\$\frac{4}{15}$ left.

19. A man owning $\frac{5}{8}$ of a mill sold $\frac{3}{5}$ of his share; how much did he still own?

20. In one rod there are $5\frac{1}{2}$ yards; how many yards are there in $3\frac{1}{3}$ rods?

21. Jane earned $\$2\frac{1}{2}$ and spent $\frac{3}{4}$ of it; how much has she left?

1.

REVIEW.

To be answered without solution.

1. Multiply $\frac{1}{3}$ by 4.
2. Multiply $\frac{3}{5}$ by 5.
3. Multiply $\frac{4}{9}$ by 6.
4. Multiply 6 by $\frac{4}{9}$.
7. Multiply $\frac{2}{5}$ by 8.
8. Multiply $\frac{3}{7}$ by 9.
9. Multiply $\frac{11}{14}$ by 7.
10. Multiply 6 by $\frac{3}{7}$.

NOTE.—Multiplying 6 by $\frac{4}{9}$ is equivalent to taking $\frac{4}{9}$ of 6.

SOLUTION.—$\frac{4}{9}$ of 6 is 4 times $\frac{1}{9}$ of 6; $\frac{1}{9}$ of 6 is $\frac{6}{9}$, and 4 times $\frac{6}{9}$ are $\frac{24}{9} = 2\frac{6}{9} = 2\frac{2}{3}$. Therefore, 6 multiplied by $\frac{4}{9}$, or $\frac{4}{9}$ of 6, equals $2\frac{2}{3}$.

5. Multiply 5 by $\frac{3}{5}$.
6. Multiply 4 by $\frac{1}{3}$.
11. Multiply 9 by $\frac{8}{11}$.
12. Multiply 11 by $\frac{3}{8}$.

2.

What is—

1. $\frac{1}{2}$ of $\frac{2}{3}$ of a dollar?
2. $\frac{2}{3}$ of $\frac{3}{7}$ of an apple?
3. $\frac{3}{5}$ of $\frac{7}{9}$ of a mile?
4. $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of a quire?
5. $\frac{3}{5}$ of $\frac{1}{3}$ of $\frac{10}{11}$ of a dollar?

What is—

6. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of a bushel?
7. $\frac{1}{2}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of a peck?
8. $\frac{2}{3}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of a dollar?
9. $\frac{1}{9}$ of $\frac{9}{10}$ of $\frac{10}{21}$ of a mile?
10. $\frac{1}{4}$ of $\frac{4}{9}$ of $\frac{9}{11}$ of a mile?

3.

1. Multiply $2\frac{1}{3}$ by $3\frac{1}{5}$.

NOTE.—Either reduce the mixed numbers to improper fractions or multiply as follows:

3 times 2 are	6	OR,
3 times $\frac{1}{5}$ are ($\frac{3}{5}$)	1	Reduce to improper fractions.
$\frac{1}{5}$ of 2 is	$\frac{2}{5}$	$2\frac{1}{3} = \frac{7}{3}$. $3\frac{1}{5} = \frac{16}{5}$.
$\frac{1}{5}$ of $\frac{1}{3}$ is	$\frac{1}{15}$	$\frac{7}{3}$ multiplied by $\frac{16}{5} = \frac{112}{15} = 7\frac{7}{15}$.
	$7\frac{7}{15}$	

SUGGESTION.—It is advisable for the pupil to practice both of the above processes.

2. Multiply $1\frac{2}{3}$ by $1\frac{5}{6}$.
3. Multiply $2\frac{1}{7}$ by $2\frac{1}{10}$.
4. Multiply $1\frac{1}{2}$ by $3\frac{1}{3}$.
5. Multiply $1\frac{1}{5}$ by $1\frac{4}{5}$.
6. Multiply $3\frac{1}{2}$ by $1\frac{5}{6}$.
7. Multiply $2\frac{1}{6}$ by $3\frac{1}{2}$.
8. Multiply $1\frac{4}{7}$ by $1\frac{1}{11}$.
9. Multiply $6\frac{1}{2}$ by $5\frac{1}{3}$.

PROBLEMS COMBINING ADDITION, SUBTRACTION AND MULTIPLICATION.

1. Mrs. Day bought $3\frac{1}{2}$ yards of ribbon at 10 cents a yard, 4 spools of thread at $4\frac{1}{2}$ cents a spool, and 2 braids at $12\frac{1}{2}$ cents each; how much did she pay for all?

SOLUTION.—She paid for all the sum of the products of $3\frac{1}{2}$ times 10 cents, 4 times $4\frac{1}{2}$ cents and 2 times $12\frac{1}{2}$ cents. $3\frac{1}{2}$ times 10¢ are 35¢, 4 times $4\frac{1}{2}$¢ are 18¢, 2 times $12\frac{1}{2}$¢ are 25¢; 35¢ and 18¢ are 53¢; 53¢ and 25¢ are 78¢. Therefore, she paid 78¢ for all.

2. Mrs. Miller bought 6 pounds of sugar at $4\frac{1}{2}$¢ a pound, 14 boxes of berries at $3\frac{1}{2}$¢ a box, and 10 pounds of currants at 5 cents a pound; how much change ought she to receive from a two-dollar bill?

3. A man traveled at the rate of $6\frac{1}{2}$ miles an hour for 7 hours, and $8\frac{3}{4}$ miles an hour for 4 hours; he then traveled on his return at the rate of 8 miles an hour for $6\frac{1}{2}$

hours, and $9\frac{1}{2}$ miles an hour for 2 hours; how far was he then from the place of starting?

4. I bought 24 handkerchiefs at $\$\frac{3}{8}$ a piece, and 2 dozen napkins at $\$1\frac{7}{8}$ a dozen; how much did I pay for all?

5. Mrs. Brown bought 6 yards of ribbon at $16\frac{2}{3}¢$ a yard, $5\frac{1}{2}$ yards of lining at $14¢$ a yard, and 4 dozen buttons at $12\frac{1}{2}¢$ a dozen. She gave in exchange for the above purchases 12 dozen eggs at $12\frac{1}{2}¢$ a dozen, and the remainder in cash; what was her cash payment?

6. A farmer has 200 trees in his orchard; $\frac{1}{5}$ of them are pear trees, $\frac{1}{4}$ of the remainder are peach trees, and the rest are apple trees; how many apple trees has he?

7. Mr. Cole bought 15 ink-stands at $\$\frac{4}{5}$ each, 8 quires of paper at $\$\frac{3}{8}$ each, and 5 books at $\$\frac{3}{4}$ each; how much more than $15 did he spend?

8. Mrs. Johnson bought a 15-pound turkey at $\$\frac{1}{6}$ per pound, and 2 quarts of cranberries at $12\frac{1}{2}¢$ a quart; she spent the rest of her money for celery; how much did she spend for celery if she spent in all $3?

9. Lucy is $9\frac{1}{4}$ years old and Mary is $5\frac{1}{2}$ years older; what is $\frac{1}{2}$ the sum of their ages?

10. Mrs. Arnold collected $\$3\frac{1}{2}$ from each of 6 people, $\$5\frac{1}{4}$ from each of 4 people; if she bought 10 pairs of shoes at $\$1\frac{1}{2}$ each, how much money has she left?

11. I spent $\frac{1}{2}$ of my income for board and clothing, $\frac{1}{4}$ of the remainder for books; $\frac{1}{3}$ of what then remained for incidentals, and saved the rest; what part of my income did I save?

12. Mr. Pringle sold 5 neckties at $\$\frac{3}{8}$ each, and 2 shirts at $\$\frac{3}{4}$ each; the purchasers owed also $\$\frac{5}{8}$ for laundry; what was the entire bill?

13. There are 2 windows that require $5\frac{1}{2}$ yards each for curtains; if each yard cost me $\$\frac{5}{8}$, how much will I have left from a ten-dollar bill?

14. In traveling 60 miles, a man traveled $\frac{3}{5}$ of the distance the first day, $\frac{1}{10}$ the second day, and the remainder the third day; how many miles did he travel the third day?

15. An aunt gave me $144. I spent $\frac{1}{12}$ of it for a cape, $\frac{1}{11}$ of the remainder for a hat and shoes, and deposited the rest in a savings bank; how much did I deposit in the bank?

16. Mr. Rice bought 2 bicycles at $100 each. He gave in exchange 4 loads of hay at $\$18\frac{1}{2}$ a load, and the remainder in cash; what was his cash payment?

17. Miss East bought 3 yards of ribbon at $\$\frac{3}{8}$ a yard, $7\frac{1}{2}$ yards at 16¢ a yard, and 8 yards at 20¢ a yard; how much less than $5 did she spend?

18. Mrs. Cleveland bought 5 handkerchiefs at $\$1\frac{1}{2}$ each, 5 at $\$1\frac{3}{4}$ each, and spent the rest of $20 for driving gloves; how much did she spend for driving gloves?

19. Martha bought 2 Christmas presents at $\$3\frac{3}{4}$ each, and 5 birthday presents at $\$1\frac{1}{5}$ each; how much less than $15 did she spend?

20. For the Infants' Hospital at Charlotte, Jane collected $\$2\frac{3}{4}$, George collected $\$3\frac{1}{2}$, Samuel collected $\$1\frac{5}{6}$; how much was collected by the three children?

21. Mr. Stout paid $\frac{1}{6}$ of his income for board, $\frac{1}{5}$ of the remainder for clothing; how much has he left, if his income is $1200?

22. In a certain school-room there are 36 pupils; if $\frac{4}{9}$ of the whole number are boys, how many boys are there? How many girls are there?

23. To the product of 3 and $4\frac{1}{2}$ add the difference between $4\frac{1}{2}$ and 3.

24. $(\frac{1}{8} \times \frac{8}{9} \times \frac{3}{8}) + (\frac{3}{4} - \frac{3}{8}) =$ what?

25. $(2\frac{1}{2} + 3\frac{1}{4}) \times (3\frac{1}{2} - 2\frac{1}{4}) =$ what?

26. $(7\frac{1}{3} - 3\frac{1}{6}) \times 2\frac{1}{2} =$ what?

27. Multiply the sum of $1\frac{1}{3}$ and $2\frac{1}{2}$ by the difference between $2\frac{1}{2}$ and $1\frac{1}{3}$.

28. What is the product of $3\frac{1}{4}$ times $3\frac{1}{2}$?

29. To the product of $\frac{3}{8}$ and $\frac{8}{9}$ add the product of $\frac{4}{5}$ and $\frac{5}{6}$.

30. $(\frac{5}{6} \times \frac{6}{7} \times \frac{7}{8}) - (\frac{3}{8} \times \frac{1}{3}) =$ what?

31. From 20 take the product of 14 times $\frac{2}{7}$.

32. Take $1\frac{1}{5}$ from the product of $7\frac{1}{2}$ times $\frac{1}{3}$.

33. Take $2\frac{1}{8}$ from the sum of $3\frac{1}{4}$ and $4\frac{1}{2}$.

34. Add $\frac{2}{3}$ and $\frac{5}{6}$, subtract $\frac{1}{4}$ from the sum and multiply the remainder by $\frac{4}{9}$.

35. Subtract $1\frac{1}{4}$ from 2, multiply the remainder by 3, and add 16 to the product.

36. From the sum of $\frac{1}{2}$ and $\frac{1}{3}$ take the sum of $\frac{1}{6}$ and $\frac{1}{8}$.

37. From the sum of $\frac{2}{3}$ and $\frac{3}{4}$ take the difference between $\frac{2}{3}$ and $\frac{3}{4}$.

38. From the difference between 6 and $1\frac{1}{6}$ take the product of $\frac{1}{2}$ and $\frac{1}{3}$.

39. From the product of $1\frac{1}{2}$ and $1\frac{1}{2}$ take the product of $\frac{1}{2}$ and $\frac{1}{2}$.

40. Which is the greater, $\frac{4}{5}$ or $\frac{5}{6}$, and how much?

41. Which is the greater, and how much, $\frac{3}{4}$ or $\frac{7}{8}$?

42. I gave \$$15\frac{1}{2}$ for a watch and had \$$10\frac{3}{4}$ left; how much money had I at first?

43. $(\frac{5}{6} + \frac{1}{2} - \frac{1}{3}) \times \overline{\frac{2}{3} \times \frac{3}{4}} - \frac{1}{3} =$ what?

44. A boy spent $\frac{3}{5}$ of his earnings for board and $\frac{1}{4}$ for clothing; what part has he left?

1.

ALIQUOT PARTS.

1. The ALIQUOT PARTS of a number are such parts as will exactly divide that number.

Thus, 5 and $7\frac{1}{2}$ are aliquot parts of 15.

2. An *aliquot part* may be a whole or mixed number, but a *factor* must be a whole number.

ALIQUOT PARTS OF $1.00.

$50\ ¢ = \$\frac{1}{2}$.	$10\ ¢ = \$\frac{1}{10}$.
$33\frac{1}{3}¢ = \$\frac{1}{3}$.	$8\frac{1}{3}¢ = \$\frac{1}{12}$.
$25\ ¢ = \$\frac{1}{4}$.	$6\frac{1}{4}¢ = \$\frac{1}{16}$.
$20\ ¢ = \$\frac{1}{5}$.	$5\ ¢ = \$\frac{1}{20}$.
$16\frac{2}{3}¢ = \$\frac{1}{6}$.	$3\frac{1}{3}¢ = \$\frac{1}{30}$.
$12\frac{1}{2}¢ = \$\frac{1}{8}$.	$2\frac{1}{2}¢ = \$\frac{1}{40}$.

$1\frac{2}{3}¢ = \$\frac{1}{60}$.

2.

1. What will 56 yards of cloth cost at 50¢ a yard?

SOLUTION 1.—At $1 a yard, the cost would be $56. But, since the price is $\$\frac{1}{2}$ a yard, the cost will be $\frac{1}{2}$ of $56, or $28. Therefore, 56 yards will cost $28.

SOLUTION 2.—Since the price is $\$\frac{1}{2}$ a yard, the cost will be $\frac{1}{2}$ as many dollars as yards $\frac{1}{2}$ of 56 is 28, or $28. Therefore, etc.

2. What will 16 dozen eggs cost at $12\frac{1}{2}¢$ a dozen?

3. What will 24 yards of cambric cost at 25¢ a yard?

4. What will 16 pounds of butter cost at 20¢ a pound?

5. At $12\frac{1}{2}$ cents a dozen, what will 15 dozen bananas cost?

6. At $16\frac{2}{3}¢$ per hour, what wages will a man earn in a day of 12 hours?

7. What will 75 pounds of coffee cost at $33\frac{1}{3}¢$ a pound?

8. At 5¢ a pound, what will a barrel of sugar containing 200 pounds cost?

9. Find the cost of 40 bunches of fire-crackers at 10¢ a bunch.

10. At $8\frac{1}{3}$ cents a yard, what will 48 yards of lining cost?

11. What will 64 pounds of sugar cost at $6\frac{1}{4}$¢ a pound?

12. At $3\frac{1}{3}$¢ each, what will 60 oranges cost?

13. What is the cost of 80 lemons, if each lemon costs $2\frac{1}{2}$ cents?

14. What is the cost of 99 bushels of oats at $33\frac{1}{3}$ cents a bushel?

15. What will 120 yards of sheeting cost at $8\frac{1}{3}$¢ a yard?

16. At 25 cents each, what will 40 salt barrels cost?

17. Find the cost of 32 spools of thread at $6\frac{1}{4}$¢ a spool.

18. At $12\frac{1}{2}$¢ a bushel, what will 56 bushels of apples cost?

19. If a pound of meat cost $16\frac{2}{3}$ cents, what will 24 pounds cost?

20. At 50¢ each, what will 16 thimbles cost?

21. What will 48 plants cost at $6\frac{1}{4}$ cents each?

22. At $3\frac{1}{3}$¢ each, what will 60 Christmas cards cost?

23. What will 40 lead pencils cost at $2\frac{1}{2}$ cents each?

24. At $1\frac{2}{3}$ cents each, what will 120 bananas cost?

25. At 25 cents a yard, what will 12 yards of ribbon cost?

26. What will 40 writing books cost at $12\frac{1}{2}$¢ apiece?

27. At $2\frac{1}{2}$ cents each, what will 80 rulers cost?

28. What will 96 books cost at $33\frac{1}{3}$¢ each?

29. When potatoes are worth $66\frac{2}{3}$¢ a bushel, how many pounds of coffee at $33\frac{1}{3}$¢ a pound will pay for 24 bushels of potatoes?

DIVISION OF FRACTIONS.

Comparison of Multiplication and Division.

I. *Multiplying* the *multiplier* by any number *multiplies* the product by the same number.

II. *Multiplying* the *divisor* by any number *divides* the quotient by the same number.

III. *Dividing* the *multiplier* by any number *divides* the product by the same number.

IV. *Dividing* the *divisor* by any number *multiplies* the quotient by the same number.

To Divide a Fraction by an Integer.

1. If 3 handkerchiefs cost \$$\frac{3}{4}$, what will one handkerchief cost?

Solution 1.—One handkerchief will cost as much as 3 is contained times in \$$\frac{3}{4}$. 1 is contained in $\frac{3}{4}$, $\frac{3}{4}$ times. 3 is contained in $\frac{3}{4}$, $\frac{1}{3}$ of $\frac{3}{4}$, or $\frac{1}{4}$ times, or \$$\frac{1}{4}$. Therefore, 1 handkerchief will cost \$$\frac{1}{4}$. Prin. II.

Or, since dividing a number by 3 is equivalent to getting $\frac{1}{3}$ of it, we may say—

Solution 2.—Since 3 handkerchiefs cost \$$\frac{3}{4}$, one handkerchief will cost $\frac{1}{3}$ of \$$\frac{3}{4}$, or \$$\frac{1}{4}$. Therefore, etc.

Question.—Dividing by 3 is the same as multiplying by what fraction?

2. Mr. Smith divided $\frac{7}{8}$ of his estate among his 6 children; what part did each receive?

3. If you divide $\frac{5}{6}$ of a barrel of flour among 4 poor families, what part of a barrel does each receive?

4. John had $\frac{3}{4}$ of a water melon which he wished to divide equally among 6 children; what part of a melon will each receive?

5. If 10 peaches cost $\$\frac{7}{8}$, what will one peach cost?

6. If 10 pencils cost $\$\frac{3}{10}$, what will one pencil cost?

7. If 20 oranges can be bought for $\$\frac{4}{5}$, what is the price of one orange?

8. What is the price of one pound of sugar if 12 pounds can be bought for $\$\frac{3}{5}$?

9. If we divide $\frac{3}{4}$ of a dozen of bananas among 6 children, what part of a dozen would each receive? How many bananas?

10. If I spent $\frac{1}{6}$ of an hour in working 5 examples, how long did I spend upon one?

11. $\frac{3}{4} \div 4$ equals how many? $\frac{3}{8} \div 5$ equals how many?

12. $\frac{6}{7} \div 6$ equals how many? $\frac{14}{15} \div 7$ equals how many?

13. $\frac{2}{3} \div 8$ equals how many? $\frac{7}{9} \div 3$ equals how many?

14. $1\frac{2}{3}$ or $\frac{5}{3}$ divided by 6 equals how many?

15. $1\frac{3}{5}$ or $\frac{8}{5}$ divided by 16 equals how many?

How, then, may we divide a fraction by an integer?

Ans. Divide the numerator or multiply the denominator of the dividend by the divisor.

To Divide an Integer by a Fraction.

1. I paid $6, which was $\frac{1}{3}$ of my money, for a table; how much money had I?

Solution.—If $6 is $\frac{1}{3}$ of my money, $\frac{3}{3}$, or the whole, is 3 times $6, or $18. Or, $6 is $\frac{1}{3}$ of 3 times $6, which are $18. Therefore, I had $18.

2. 13 bushels is $\frac{1}{6}$ of what number of bushels?

3. John spent 12 cents, which was $\frac{1}{8}$ of his money; how much money had he?

4. Nine acres comprise $\frac{1}{11}$ of my farm; how many acres in the whole farm?

5. Thomas has worked 6 examples; if he has finished $\frac{1}{4}$ of his lesson, how many examples in the whole lesson?

6. If $\frac{2}{3}$ of a barrel of flour cost \$6, what will 1 barrel cost?

SOLUTION 1.—If $\frac{2}{3}$ of a barrel cost \$6, or $\$\frac{18}{3}$, 1 barrel will cost as many dollars as $\frac{2}{3}$ is contained times in $\frac{18}{3}$, or 9 times, or \$9. Therefore, etc.

SOLUTION 2.—If $\frac{2}{3}$ of a barrel cost \$6, $\frac{1}{3}$ of a barrel will cost $\frac{1}{2}$ of \$6, or \$3; $\frac{3}{3}$, or 1 barrel, will cost 3 times \$3, or \$9. Therefore, etc.

7. Mr. Stewart spent \$15 for a coat; if the sum spent for a coat was $\frac{3}{10}$ of his money, how much money had he?

8. Mr. Blair spent $\frac{3}{11}$ of his money, or \$1500, in furnishing a house; how much money had he?

9. If a boy can earn $\$\frac{4}{5}$ in a day, how long will it take him to earn \$12?

SOLUTION.—As many days as $\$\frac{4}{5}$ is contained times in \$12; 1 is contained in 12, 12 times; $\frac{1}{5}$ is contained in 12, 5 times as many times as 1, or 5 times 12 times, or 60 times; $\frac{4}{5}$ is contained in 12 one-fourth of 60 times, or 15 times, or 15 days. Therefore, etc. Or, like solution 1, problem 6.

10. If $\frac{4}{5}$ of a ton of hay cost \$8, what will 1 ton cost?

11. If 1 apple can be bought for $\frac{3}{4}$¢, how many can be bought for 21¢?

12. If $\frac{9}{10}$ of a house cost \$2700, what did the whole house cost?

13. If $\frac{8}{9}$ of the length of a kite string is 320 feet, what is the length of the entire string?

14. If 140, or $\frac{7}{12}$, of the whole number of pupils in a school are boys, how many pupils in the school?

15. If $\frac{5}{9}$ of a ship is valued at \$15000, what is the value of the entire ship?

NOTE.—When the solutions given above are thoroughly understood, the following may be used:

SOLUTION.—If $\frac{5}{9}$ of the ship be valued at \$15000, $\frac{9}{9}$, or the whole ship, will be valued at 9 times $\frac{1}{9}$ of \$15000. $\frac{1}{9}$ of \$15000 is \$3000, and 9 times \$3000 are \$27000. Therefore, the value of the ship is \$27000.

16. Mr. Barrows spent \$12, which was $\frac{4}{7}$ of his week's salary; what is his week's salary?

17. How many months in a year, if in $\frac{3}{4}$ of a year there are 9 months?

18. A boy gave away 15 marbles; if this was $\frac{5}{12}$ of the whole number he had, how many had he?

19. Mr. Mills gave \$90 for a buggy; if this sum was $\frac{9}{20}$ of what he gave for a horse, how much did he give for a horse?

20. If a train move at the rate of $\frac{5}{6}$ of a mile in a minute, how long will it be in running 50 miles?

21. How many pounds of butter, worth \$$\frac{3}{10}$ a pound, will pay for \$6 worth of flour?

22. If $\frac{4}{7}$ of a barrel of sugar cost \$8, what will one barrel cost?

23. $42 \div \frac{3}{7}$ = what? $108 \div \frac{9}{11}$ = what?

24. $36 \div \frac{8}{8}$ = what? $80 \div \frac{10}{13}$ = what?

25. $16 \div 1\frac{1}{7}$ (or $\frac{8}{7}$) = what? $56 \div \frac{14}{15}$ = what?

How, then, may we divide an integer by a fraction?

Ans. Invert the terms of the divisor and proceed as in multiplication of fractions.

TO DIVIDE A FRACTION BY A FRACTION.

1. If \$$\frac{2}{3}$ will buy 1 yard of cloth, how much can be bought for \$$\frac{3}{4}$?

SOLUTION 1.—As many yards can be bought for $\$\frac{2}{4}$ as $\$\frac{3}{8}$ or $\$\frac{3}{12}$ is contained times in $\$\frac{2}{4}$ or $\$\frac{9}{12}$, which is $1\frac{1}{3}$ times or $1\frac{1}{3}$ yards. Therefore, etc.

SOLUTION 2.—As many yards as $\$\frac{3}{8}$ is contained times in $\$\frac{2}{4}$. 1 is contained in $\frac{2}{4}$, $\frac{2}{4}$ times; $\frac{1}{3}$ is contained in $\frac{2}{4}$, 3 times as many times as 1, or 3 times $\frac{2}{4}$, or $\frac{9}{4}$ times. $\frac{3}{8}$ is contained in $\frac{2}{4}$, $\frac{1}{3}$ of $\frac{9}{4}$ times, or $\frac{9}{8}$ times, which equals $1\frac{1}{3}$ times, or $1\frac{1}{3}$ yards. Therefore, etc.

2. At $\$\frac{3}{5}$ a pound, how much tea can be bought for $\$\frac{7}{8}$?

3. If $\frac{2}{3}$ of a yard of cloth cost $\$\frac{7}{8}$, what will 1 yard cost?

4. If $\frac{2}{7}$ of a drum of figs cost $\$\frac{5}{9}$, what will 1 drum cost?

5. What will 1 dozen oranges cost, if $\frac{5}{6}$ of a dozen cost $\$\frac{3}{10}$?

6. What will 1 dozen eggs cost, if $\frac{2}{9}$ of a dozen cost $\$\frac{1}{25}$?

7. What is the value of a clock, if $\frac{4}{9}$ of its value is $\$1\frac{1}{9}$?

8. If I can do $\frac{4}{5}$ of a piece of work in $\frac{3}{8}$ days, how long will it take me to do the whole work?

9. How many ink wells at $\$1\frac{1}{8}$ each can be bought for $\$4\frac{1}{2}$?

NOTE.—Reduce mixed numbers to improper fractions.

10. How many books at $\$2\frac{1}{2}$ each can be bought for $\$17\frac{1}{2}$?

11. If a boy can earn $\$\frac{9}{10}$ in $\frac{3}{4}$ of a day, how much can he earn in 1 day?

12. If $\frac{4}{5}$ of the value of a couch is $\$14\frac{2}{5}$, what is the full value?

13. If a man can walk $27\frac{1}{2}$ miles in $5\frac{1}{2}$ hours, how far can he walk in 1 hour?

14. Irene gave $\frac{3}{4}$ of a month's salary for a cloak; if she paid $\$33\frac{3}{4}$ for the cloak, what is her salary per month?

15. How many gallons of gasoline at $\$\frac{3}{20}$ a gallon can be bought for $\$2\frac{1}{3}$?

16. If a yard of muslin can be bought for $\frac{2}{3}$ of a dime, how much can be bought for $\frac{5}{6}$ of a dime?

17. If $\frac{3}{4}$ of a yard of velvet can be bought for \$$2\frac{1}{4}$, what must be paid for one yard?

18. If $\frac{3}{7}$ of a yard of velvet can be bought for \$$1\frac{1}{2}$, how much must be paid for one yard?

19. If $\frac{2}{5}$ of a yard of plush can be bought for \$$3\frac{1}{2}$, how much will one yard cost?

20. If $3\frac{1}{2}$ pounds of pulverized sugar cost $24\frac{1}{2}$¢, how much will one pound cost?

21. If $1\frac{2}{5}$ yards of lace cost $17\frac{1}{2}$¢, what will one yard cost?

22. If $1\frac{2}{7}$ tons of hay cost \$$22\frac{1}{2}$, what will one ton cost?

23. How many times is $\frac{3}{4}$ contained in $\frac{5}{6}$?

SOLUTION.—$\frac{3}{4}$ is equal to $\frac{9}{12}$, and $\frac{5}{6}$ is equal to $\frac{10}{12}$: $\frac{9}{12}$ is contained times in $\frac{10}{12}$, $1\frac{1}{9}$ times. Therefore, $\frac{3}{4}$ is contained in $\frac{5}{6}$, $1\frac{1}{9}$ times.

24. How many times is $4\frac{1}{3}$ contained in $6\frac{1}{2}$?

25. How many times is $\frac{6}{7}$ contained in $\frac{3}{5}$?

26. How many times is $\frac{2}{3}$ contained in $\frac{13}{15}$?

27. How many times is $1\frac{2}{3}$ contained in $7\frac{2}{5}$?

28. How many times is $1\frac{1}{4}$ contained in $2\frac{1}{2}$?

How, then, may we divide a fraction by a fraction?

Ans. Invert the divisor, and proceed as in multiplication of fractions.

PROBLEMS COMBINING ADDITION, SUBTRACTION, MULTIPLICATION AND DIVISION.

1. One-fourth of a flock of sheep are in one field, $\frac{1}{8}$ in another, $\frac{1}{6}$ in another, and the remainder, or 22 sheep, in a fourth field; how many sheep in the flock?

2. Charles is $\frac{1}{3}$ as old as his mother and $\frac{8}{9}$ as old as his brother; if Charles is 16 years old, what is the sum of his mother's age and his brother's age?

3. George gave away $\frac{2}{3}$ of his marbles and lost $\frac{3}{4}$ of the remainder; if he had left 5 marbles, how many had he at first?

4. Mr. Cole gave $\frac{2}{3}$ of his money for a cow; he gave $25 for hens; the sum spent for hens was $\frac{5}{8}$ of all the money he had left; how much had he at first?

5. Enid is 20 years old; if we subtract 4 years from her age she would be $\frac{3}{4}$ as old as her brother; what is the sum of their ages?

6. If $\frac{2}{3}$ of $\frac{6}{7}$ of a ton of coal cost $\$1\frac{3}{7}$, what will $\frac{3}{4}$ of $\frac{4}{5}$ of 27 tons cost?

7. A man engaging in play lost $\frac{1}{4}$ of his money and found that $81 was $\frac{9}{10}$ of what he had left; how much had he at first?

8. A boy being asked his age, replied that if to $\frac{3}{4}$ of his age 8 years be added, $\frac{4}{5}$ of the sum would be 16 years; how old was that remarkable boy?

9. Bought 20 pounds of sugar at $4\frac{3}{4}$¢ a pound, and 4 pounds of dates at $12\frac{1}{2}$¢ a pound; how many pounds of codfish at $14\frac{1}{2}$¢ a pound will pay for his purchases?

10. A man being asked how much money he had, replied that if from the whole sum $\frac{3}{10}$ of it be subtracted, $\frac{3}{4}$ of $\frac{4}{5}$ of the remainder would be $700; how much money had he?

11. $\frac{5}{9}$ of the sum of two numbers is 30; if one of the numbers is $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of 36, what is the other number?

12. If $\frac{3}{10}$ of the whole number of trees in an orchard are cherry trees, $\frac{2}{5}$ are plum trees, and the remainder, or 36, are peach trees, how many trees are there in the

orchard? How many cherry trees? How many plum trees?

13. How many pigs can I buy for \$26 at the rate of 3 for \$7, and have \$5 left?

14. How many barrels of flour at \$$6\frac{1}{4}$ per barrel must be given in exchange for 20 barrels of apples at \$$3\frac{1}{2}$ a barrel, and 10 bushels of oats at \$$\frac{1}{2}$ a bushel?

15. How many yards of ribbon at $16\frac{1}{2}$¢ a yard must be given for 5 hats at \$$3\frac{1}{4}$ each, and 2 pairs of shoes at \$$4\frac{1}{4}$ each?

16. If $\frac{5}{9}$ of the value of my butter is \$10, and I receive in exchange for it 100 pounds of sugar at $5\frac{1}{2}$¢ a pound, 150 bars of soap at 5¢ a bar, and the rest in money, how much money do I receive?

17. If to $\frac{3}{8}$ of $\frac{8}{9}$ of a number, $\frac{1}{4}$ of $\frac{4}{7}$ of 14 be added, $\frac{3}{5}$ of the sum will be $7\frac{1}{5}$; find the number.

18. Grace is $16\frac{1}{2}$ years old and Reuben is $6\frac{3}{4}$ years younger; if to the sum of their ages $3\frac{3}{4}$ years be added, the sum will be $\frac{6}{11}$ of their mother's age; how old is their mother?

19. A post stands $\frac{1}{6}$ in the mud, $\frac{1}{4}$ in the water, and the remainder, which is $\frac{4}{7}$ of 49 feet above the water; what is the height of the pole?

20. I earned \$60 one month and \$50 the next; if $\frac{1}{11}$ of the sum was $\frac{2}{15}$ of what I paid for a bicycle, how much did I pay for the bicycle?

21. A man spent \$$15\frac{1}{2}$ one month and \$$16\frac{1}{2}$ the next month; one-half of the sum spent would pay for how many week's board at \$4 a week?

22. How many chairs at \$$4\frac{1}{2}$ each would be equivalent in value to 9 stoves at \$30 each?

23. Mr. Snell bought 6 chairs at \$$10\frac{1}{2}$ each; he paid

at one time \$$31\frac{1}{4}$ and at another \$$13\frac{3}{4}$; how many desks at \$5 each could be bought with what he still owed?

24. If 6 pounds of soda cost \$1, what will 10 pounds cost?

25. If $\frac{4}{5}$ of what I paid for 4 pictures is 80 cents, how many pictures can I buy for $\frac{3}{5}$ of \$10?

Answers to be given without solutions.

Divide—	Divide—	Divide—	Divide—
26. $\frac{1}{3}$ by 2.	*28.* 8 by $\frac{2}{3}$.	*30.* $\frac{3}{4}$ by $\frac{4}{5}$.	*32.* $\frac{3}{8}$ by $\frac{5}{7}$.
27. $\frac{3}{4}$ by 5.	*29.* 12 by $\frac{4}{5}$.	*31.* $\frac{1}{2}$ by $2\frac{1}{2}$.	*33.* $4\frac{1}{2}$ by $2\frac{1}{3}$.

34. Add $\frac{1}{3}$ and $\frac{1}{2}$, multiply the sum by $\frac{1}{5}$, from the product subtract $\frac{1}{8}$, divide the remainder by $\frac{1}{6}$, and what is the result?

35. Divide $4\frac{1}{6}$ by $\frac{5}{12}$, add $2\frac{1}{2}$ to the quotient, multiply the sum by $\frac{1}{5}$, and subtract 5 from the product; what is the remainder?

36. Find $\frac{4}{5}$ of 45, divide the product by 6, to the quotient add $5\frac{1}{4}$, and what is the sum?

37. $(3\frac{1}{2} + 1\frac{3}{4}) \div (\frac{3}{4} + \frac{3}{8}) \times 1\frac{2}{7}$ equal what?

38. From 18 take $10\frac{2}{3}$, to the remainder add $5\frac{1}{6}$, multiply the sum by 6, and divide the product by $\frac{1}{2}$; what is the result?

39. Multiply 56 by $\frac{3}{8}$, divide the product by $\frac{7}{10}$, subtract $18\frac{1}{3}$ from the quotient, and what is the result?

40. 3 times $(1\frac{1}{2} + 2\frac{1}{3})$ divided by $\frac{5}{18}$ equals what?

41. $5\frac{1}{6} - 3\frac{2}{3} + 8\frac{1}{4} - 3$ equals what?

42. ($\frac{3}{4}$ of 24) $\div$ ($\frac{1}{3} \div 6$) + 14 equals what?

43. $(9\frac{1}{4} + 8\frac{3}{4}) \div \frac{3}{5} \times \frac{19}{12} \times 12 =$ what?

44. Multiply $3\frac{1}{2}$ by $3\frac{1}{2}$ by reducing the mixed numbers to improper fractions before multiplying.

45. To the product of 64 times $\frac{7}{8}$ add the difference between $3\frac{1}{4}$ and $2\frac{1}{2}$; what is the result?

To Find a Number, when a Fractional Part of It is Given.

1.

1. 15 is $\frac{1}{3}$ of what number?

Solution.—Since 15 is $\frac{1}{3}$ of some number, $\frac{3}{3}$, or the whole number, must be 3 times 15, or 45. Therefore, 15 is $\frac{1}{3}$ of 45.

Or, 15 is $\frac{1}{3}$ of 3 times 15, or 45. Therefore, etc.

2. 8 is $\frac{1}{5}$ of what number?

3. 7 is $\frac{1}{11}$ of what number?

4. 16 is $\frac{1}{3}$ of what number?

5. 12 is $\frac{1}{12}$ of what number?

6. 6 is $\frac{1}{16}$ of what number?

2.

1. 18 is $\frac{3}{4}$ of what number?

Solution.—Since 18 is $\frac{3}{4}$ of some number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of 18, or 6, and $\frac{4}{4}$, or the whole number, must be 4 times 6, or 24. Therefore, 18 is $\frac{3}{4}$ of 24.

Or, 18 is $\frac{3}{4}$ of 4 times $\frac{1}{3}$ of 18. $\frac{1}{3}$ of 18 is 6, and 4 times 6 are 24. Therefore, etc.

2. 21 is $\frac{3}{7}$ of what number?

3. 27 is $\frac{9}{10}$ of what number?

4. 64 is $\frac{8}{9}$ of what number?

5. 72 is $\frac{9}{11}$ of what number?

6. 49 is $\frac{7}{8}$ of what number?

7. 132 is $\frac{6}{7}$ of what number?

8. 120 is $\frac{2}{13}$ of what number?

9. 99 is $\frac{9}{10}$ of what number?

10. 56 is $\frac{8}{11}$ of what number?

11. 20 is $\frac{10}{17}$ of what number?

3.

1. $\frac{2}{3}$ is $\frac{1}{4}$ of what number?

Solution.—$\frac{2}{3}$ is $\frac{1}{4}$ of 4 times $\frac{2}{3}$, or $\frac{8}{3}=2\frac{2}{3}$. Therefore, $\frac{2}{3}$ is $\frac{1}{4}$ of $2\frac{2}{3}$.

2. $\frac{2}{3}$ is $\frac{3}{4}$ of what number?

SOLUTION.—Since $\frac{2}{3}$ is $\frac{3}{4}$ of some number, $\frac{1}{4}$ of the number is $\frac{1}{3}$ of $\frac{2}{3}$, or $\frac{2}{9}$, and $\frac{4}{4}$, or the whole number, must be 4 times $\frac{2}{9}$, or $\frac{8}{9}$. Therefore, $\frac{2}{3}$ is $\frac{3}{4}$ of $\frac{8}{9}$.

Or, $\frac{2}{3}$ is $\frac{3}{4}$ of 4 times $\frac{1}{3}$ of $\frac{2}{3}$. $\frac{1}{3}$ of $\frac{2}{3}$ is $\frac{2}{9}$, and 4 times $\frac{2}{9}$ are $\frac{8}{9}$. Therefore, $\frac{2}{3}$, etc.

3. $\frac{3}{4}$ is $\frac{1}{9}$ of what number?
4. $\frac{5}{6}$ is $\frac{3}{4}$ of what number?
5. $\frac{1}{2}$ is $\frac{5}{6}$ of what number?
6. $\frac{3}{7}$ is $\frac{1}{8}$ of what number?
7. $\frac{8}{9}$ is $\frac{13}{27}$ of what number?
8. $\frac{4}{9}$ is $\frac{2}{3}$ of what number?

4.

1. $3\frac{1}{2}$ is $\frac{4}{5}$ of what number?

NOTE.—Reduce mixed numbers to improper fractions.

SOLUTION.—Since $\frac{7}{2}$ is $\frac{4}{5}$ of some number, $\frac{1}{5}$ of the number is $\frac{1}{4}$ of $\frac{7}{2}$, or $\frac{7}{8}$, and $\frac{5}{5}$, or the number, must be 5 times $\frac{7}{8}$, or $\frac{35}{8} = 4\frac{3}{8}$. Therefore, $3\frac{1}{2}$ is $\frac{4}{5}$ of $4\frac{3}{8}$.

Or, $\frac{7}{2}$ is $\frac{4}{5}$ of 5 times $\frac{1}{4}$ of $\frac{7}{2}$. $\frac{1}{4}$ of $\frac{7}{2}$ is $\frac{7}{8}$, and 5 times $\frac{7}{8}$ are $\frac{35}{8} = 4\frac{3}{8}$. Therefore, etc.

2. $3\frac{1}{6}$ is $\frac{2}{3}$ of what number?
3. $2\frac{1}{7}$ is $\frac{5}{14}$ of what number?
4. $4\frac{1}{6}$ is $\frac{5}{7}$ of what number?
5. $1\frac{2}{3}$ is $\frac{3}{7}$ of what number?
6. $5\frac{1}{4}$ is $\frac{7}{9}$ of what number?

5.

1. 16 is $\frac{1}{2}$ of $\frac{2}{3}$ of what number?

SOLUTION.—$\frac{1}{2}$ of $\frac{2}{3}$ is $\frac{1}{3}$. The question now is, 16 is $\frac{1}{3}$ of what number? (See former solution.)

2. 3 is $\frac{1}{9}$ of $\frac{9}{10}$ of what number?
3. 15 is $\frac{1}{7}$ of $\frac{7}{10}$ of what number?

4. 7 is $\frac{1}{11}$ of $\frac{11}{12}$ of what number?
5. 14 is $\frac{1}{5}$ of $\frac{5}{7}$ of what number?
6. 8 is $\frac{1}{6}$ of $\frac{6}{11}$ of what number?
7. 45 is $\frac{1}{2}$ of $\frac{2}{3}$ of what number?
8. 22 is $\frac{1}{4}$ of $\frac{4}{5}$ of what number?
9. 10 is $\frac{2}{3}$ of $\frac{1}{2}$ of what number?
10. 15 is $\frac{5}{7}$ of $\frac{7}{6}$ of what number?

6.

1. In $\frac{3}{4}$ of 60, how many times 5?

SOLUTION.—$\frac{3}{4}$ of 60 equals 3 times $\frac{1}{4}$ of 60. $\frac{1}{4}$ of 60 is 15, and 3 times 15 are 45. 45 is as many times 5 as 5 is contained times in 45, which is 9 times. Therefore, $\frac{3}{4}$ of 60 is 9 times 5.

2. $\frac{2}{3}$ of 45 is how many times 3?
3. $\frac{4}{5}$ of 40 is how many times 16?
4. $\frac{5}{6}$ of 48 is how many times 4?
5. $\frac{6}{7}$ of 49 is how many times 8?
6. $\frac{7}{8}$ of 56 is how many times 10?
7. $\frac{8}{9}$ of 81 is how many times 12?
8. $\frac{9}{10}$ of 90 is how many times 27?
9. $\frac{10}{11}$ of 88 is how many times 10?
10. $\frac{11}{12}$ of 120 is how many times 9?
11. $\frac{5}{9}$ of 36 is how many times 3?
12. $\frac{11}{13}$ of 130 is how many times 9?

7.

1. 48 is $\frac{3}{5}$ of how many times 9?

SOLUTION.—48 is $\frac{3}{5}$ of 5 times $\frac{1}{3}$ of 48. $\frac{1}{3}$ of 48 is 16, and 5 times 16 are 80. 80 is as many times 9 as 9 is contained times in 80, which is $8\frac{8}{9}$ times. Therefore, 48 is $\frac{3}{5}$ of $8\frac{1}{9}$ times 9.

2. 54 is $\frac{6}{7}$ of how many times 8?
3. 36 is $\frac{9}{10}$ of how many times 5?
4. 24 is $\frac{12}{13}$ of how many times 4?

5. 81 is $\frac{9}{11}$ of how many times 12?
6. 72 is $\frac{8}{9}$ of how many times 3?
7. 16 is $\frac{4}{5}$ of how many times 15?
8. 132 is $\frac{11}{12}$ of how many times 8?
9. 121 is $\frac{11}{4}$ of how many times 4?
10. 25 is $\frac{5}{9}$ of how many times 3?

8.

1. $\frac{5}{9}$ of 90 is $\frac{5}{8}$ of how many times 11?

SOLUTION.—$\frac{5}{9}$ of 90 is 5 times $\frac{1}{9}$ of 90. $\frac{1}{9}$ of 90 is 10, and 5 times 10 are 50. 50 is $\frac{5}{8}$ of 8 times $\frac{1}{8}$ of 50. $\frac{1}{8}$ of 50 is 10, and 8 times 10 are 80. 80 is as many times 11 as 11 is contained times in 80, which is $7\frac{3}{11}$ times. Therefore, $\frac{5}{9}$ of 90 is $\frac{5}{8}$ of $7\frac{3}{11}$ times 11.

2. $\frac{3}{4}$ of 16 is $\frac{6}{7}$ of how many times 7?
3. $\frac{6}{7}$ of 42 is $\frac{9}{10}$ of how many times 5?
4. $\frac{5}{6}$ of 60 is $\frac{10}{11}$ of how many times 8?
5. $\frac{10}{11}$ of 44 is $\frac{5}{6}$ of how many times 16?
6. $\frac{2}{3}$ of 18 is $\frac{12}{13}$ of how many times 2?
7. $\frac{9}{11}$ of 33 is $\frac{3}{4}$ of how many times 11?
8. $\frac{4}{7}$ of 35 is $\frac{4}{5}$ of how many times 4?
9. $\frac{7}{11}$ of 77 is $\frac{7}{8}$ of how many times 4?

9.

1. $\frac{3}{7}$ of 35 is $\frac{5}{8}$ of how many tenths of 120?

SOLUTION.—$\frac{3}{7}$ of 35 is 3 times $\frac{1}{7}$ of 35. $\frac{1}{7}$ of 35 is 5, and 3 times 5 are 15; 15 is $\frac{5}{8}$ of 8 times $\frac{1}{8}$ of 15. $\frac{1}{8}$ of 15 is 3, and 8 times 3 are 24. 24 is as many tenths of 120 as $\frac{1}{10}$ of 120, or 12, is contained times in 24, or 2 times. Therefore, $\frac{3}{7}$ of 35 is $\frac{5}{8}$ of 2 tenths of 120.

2. $\frac{2}{5}$ of 50 is $\frac{4}{9}$ of how many thirds of 9?
3. $\frac{2}{9}$ of 63 is $\frac{7}{8}$ of how many halves of 8?
4. $\frac{5}{7}$ of 49 is $\frac{7}{9}$ of how many thirds of 15?
5. $\frac{8}{11}$ of 33 is $\frac{3}{4}$ of how many sixteenths of 64?
6. $\frac{5}{8}$ of 64 is $\frac{10}{11}$ of how many fourths of 16?
7. $\frac{9}{10}$ of 100 is $\frac{3}{4}$ of how many twentieths of 360?

1.

Review Problems in Fractions.

1. A farmer sold $\frac{3}{7}$ of his sheep and had 80 left; how many sheep had he at first?

2. I sold a carriage for $75, which was $\frac{5}{8}$ of what it cost; what was the loss?

3. By selling my bicycle for $60, I gained $\frac{3}{7}$ of its cost; what was its cost?

Solution.—$\frac{7}{7}$, the cost, plus $\frac{3}{7}$, the gain, equals $\frac{10}{7}$ of the cost, or the selling price, which is $60; $60 is $\frac{10}{7}$ of 7 times $\frac{1}{10}$ of $60; $\frac{1}{10}$ of $60 is $6, and 7 times $6 are $42. Therefore, the bicycle cost $42.

4. If 3 yards of cloth cost $\$\frac{15}{16}$, what will 7 yards cost?

5. If 4 yards of cloth cost $\$\frac{7}{9}$, what will 12 yards cost?

Note.—12 yards are 3 times 4 yards.

6. If a man spend $\$\frac{3}{8}$ a day for cigars, in what time will he spend $75?

7. A miller owns $\frac{5}{6}$ of a mill and sells $\frac{2}{3}$ of his share; what part of the mill does he still own?

Solution.—Since he sells $\frac{2}{3}$ of his share, he will have left $\frac{3}{3} - \frac{2}{3}$, or $\frac{1}{3}$ of his share. $\frac{1}{3}$ of $\frac{5}{6}$ equals $\frac{5}{18}$. Therefore, he will still own $\frac{5}{18}$ of the mill.

Note.—If a person sells $\frac{2}{3}$ of anything, he will have $\frac{1}{3}$ of it left. If he sells $\frac{5}{6}$ of anything, he will have $\frac{1}{6}$ of it left.

The reasoning is the same if he owns one whole thing or a fractional part of a thing.

8. A man sold $\frac{3}{7}$ of his farm and had 64 acres left; how many acres had he at first?

9. The difference between $\frac{5}{6}$ and $\frac{3}{5}$ of a certain number is 14; what is the number?

10. When Harry is $\frac{2}{5}$ older than he now is, he will be 21 years of age; how old is he?

11. $\frac{6}{7}$ is $\frac{3}{8}$ of what number?

12. A man is 45 years old and $\frac{2}{5}$ of his age is $\frac{3}{7}$ of the age of his wife; how old is his wife?

13. A man sold a horse for \$90, which was $\frac{1}{5}$ more than it cost him; what did the horse cost him?

NOTE.—$\frac{5}{5}$ equals the cost of the horse.

14. A lady paid \$30 for a cloak, which was $\frac{3}{7}$ more than she paid for a dress; what did she pay for the dress?

15. A, B and C together own a yacht. A owns $\frac{1}{5}$ of it, and B $\frac{2}{3}$ of it. What part does C own?

16. How much less than \$10 will 7 pounds of tea cost at \$$\frac{4}{5}$ a pound?

17. A man sold $\frac{3}{8}$ of his farm and had 100 acres left; how many acres had he at first?

18. Paid \$50 for a harness, and $\frac{3}{5}$ the cost of the harness was $\frac{3}{10}$ the cost of the horse; what was the cost of both?

19. Name four fractions that can be changed to 24ths.

20. From a piece of silk containing $9\frac{1}{2}$ yards, $3\frac{3}{4}$ yards were cut; how many yards remained?

21. Multiplying by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, etc., is the same as dividing by what integers?

22. A boy having \$$\frac{7}{8}$ gave \$$\frac{2}{5}$ for a knife; how much money had he left?

23. If $\frac{5}{7}$ be added to a certain fraction, the sum will be $\frac{9}{10}$; what is the fraction?

24. If $\frac{2}{3}$ be subtracted from a certain fraction, the remainder will be $\frac{2}{15}$; what is the fraction?

25. How much more than \$5 will 9 yards of cloth cost at \$$\frac{7}{8}$ a yard?

26. The product of two fractions is $\frac{1}{2}$; one of them is $\frac{2}{3}$; what is the other?

2.

1. If A can do a piece of work in 3 days and B can do the same work in 4 days, how long will it take them working together?

SOLUTION.—If A can do the work in 3 days, he can do $\frac{1}{3}$ of it in one day; and if B can do it in 4 days, he can do $\frac{1}{4}$ of it in one day; together they can do the sum of $\frac{1}{3}$ and $\frac{1}{4}$, or $\frac{7}{12}$, of it in one day; it will take them as long to do $\frac{12}{12}$, or the whole, as $\frac{7}{12}$ is contained times in $\frac{12}{12}$, which is $1\frac{5}{7}$ times, or $1\frac{5}{7}$ days. Therefore, it will take them $1\frac{5}{7}$ days to do the work together.

2. If A and B together can do a piece of work in 5 days, and A alone can do it in 7 days, in how many days can B do it alone?

SOLUTION.—If A and B together can do the work in 5 days, they can do $\frac{1}{5}$ of it in one day. If A can do it in 7 days, he can do $\frac{1}{7}$ of it in 1 day, and B can do the difference between $\frac{1}{5}$ and $\frac{1}{7}$, or $\frac{2}{35}$, of the work in one day, it will take him as long to do $\frac{35}{35}$, or the whole work, as $\frac{2}{35}$ is contained times in $\frac{35}{35}$, which is $17\frac{1}{2}$ times, or $17\frac{1}{2}$ days. Therefore, it will take B $17\frac{1}{2}$ days to do the work alone.

3. A can do a piece of work in 6 days and B can do the same work in 8 days, in how many days can both do it, working together?

4. B can do a piece of work in $4\frac{1}{2}$ days and C can do the same work in 6 days; how long will it take them working together?

EXPLANATION.—The fractional part that represents what A can do in one day is usually expressed by the reciprocal of the fraction or the fraction inverted.

Thus, if A can do a piece of work in 4 days, he can do $\frac{1}{4}$ of it in 1 day. If A can do a piece of work in $4\frac{1}{2}$ or $\frac{9}{2}$ days, he can do $\frac{1}{\frac{9}{2}}$ or $\frac{2}{9}$ of it in one day.

If A can do $\frac{1}{2}$ as much work as B in one day, B can do $\frac{2}{1}$, or twice as much as A in one day.

If A can do $\frac{1}{3}$ as much work as B in one day, B can do $\frac{3}{1}$, or three times as much as A in one day.

If A can do $\frac{2}{3}$ as much work as B in one day, B can do $\frac{3}{2}$ or $1\frac{1}{2}$ times as much as A in one day.

5. A, B and C can do a piece of work in 4 days; B and C can do the same work in 6 days; how long will it take A alone to do the work?

6. One pipe will fill a cistern in $3\frac{1}{2}$ hours; another in $5\frac{1}{2}$ hours; how long will it take both to fill it?

7. One pipe will fill a cistern in 2 hours, another in 3 hours, and still another in 4 hours; how long will it take the three pipes to fill it?

8. Two pipes together fill a cistern in 4 hours; the first can fill it in 6 hours; how long will it take the other to fill it?

9. Three pipes fill a cistern in $1\frac{1}{2}$ hours; two of them fill it in $2\frac{1}{4}$ hours; how long will it take the other to fill it?

10. Two pipes carry water into a tank and a third carries water from it; the first pipe will fill it in 3 hours, the second in 4 hours; the third will empty it in 2 hours. If the tank is empty and the three pipes are used, how long will it take to fill the tank?

11. If $3\frac{1}{2}$ yards of cloth cost \$14, what will $5\frac{1}{4}$ yards cost?

SOLUTION.—If $3\frac{1}{2}$ ($\frac{7}{2}$) yards cost \$14, one yard will cost 2 times $\frac{1}{7}$ of \$14. $\frac{1}{7}$ of \$14 is \$2, and 2 times \$2 are \$4. $5\frac{1}{4}$ yards, or $\frac{21}{4}$ yards, will cost 21 times $\frac{1}{4}$ of \$4. $\frac{1}{4}$ of \$4 is \$1, and 21 times \$1 are \$21. Therefore, $5\frac{1}{4}$ yards will cost \$21.

12. If $2\frac{1}{2}$ yards cost \$10, what will $1\frac{5}{9}$ yards cost?

3.

1. If Mr. Jones had \$60, and spent $\frac{2}{3}$ of it at one time and $\frac{1}{10}$ of the remainder at another, how many dollars had he left?

2. By selling a horse at a loss of \$30, I lost $\frac{3}{10}$ of its value; what was its value?

3. Mr. Elder is 50 years of age and his son is $\frac{3}{10}$ as old; what is $\frac{1}{5}$ of the sum of their ages?

4. Mr. Bowman sold a cow for \$48, which was $\frac{3}{4}$ of what he paid for her; what did he pay for her and what did he lose by the sale?

5. How many dozen eggs at \$$\frac{1}{5}$ a dozen will pay for $42\frac{1}{2}$ pounds of meat at \$$\frac{1}{6}$ per pound?

6. One and one-fourth times the sum of two equal numbers is 75; what are the numbers?

7. If a train runs 50 miles in an hour, how far will it run in $7\frac{1}{2}$ hours?

8. A man spent $\frac{3}{4}$ of \$48 for Long Branch rockers, valued at \$2 each; how many did he buy?

9. William divided $\frac{4}{9}$ of 81 marbles equally among 6 boys; how many marbles did he give to each boy?

10. Mr. Arnold gave $\frac{2}{3}$ of his money for a house; if he had \$12000, and the sum he paid for the house was $\frac{8}{9}$ of its original value, what was its original value?

11. A man having \$80 gave $\frac{4}{5}$ of it for $\frac{2}{3}$ of the value of a horse; what was the value of the horse?

4.

1. If I buy turkeys at the rate of 4 for \$3 and sell them at the rate of 9 for \$8, how much will I gain on 36 turkeys?

SUGGESTION.—Divide the price by the quantity to find the cost of one.

SOLUTION.—If I buy them at the rate of 4 for \$3, each turkey will cost \$$\frac{3}{4}$; if I sell them at the rate of 9 for \$8, each turkey will sell for \$$\frac{8}{9}$. I will gain on each turkey the difference between \$$\frac{8}{9}$ and \$$\frac{3}{4}$, which is \$$\frac{5}{36}$. On 36 turkeys I will gain 36 times \$$\frac{5}{36}$, or \$5. Therefore, I will gain \$5 on 36 turkeys.

2. If I buy oranges at the rate of 3 for 5 cents and sell them at the rate of 9 for 10 cents, do I gain or lose, and how much, on 30 oranges?

3. If I buy lemons at the rate of 7 for 6 cents and sell them at the rate of 7 for 8 cents, how much do I gain on 21 lemons?

4. George is 20 years old; if 5 years be added to his age he will be $\frac{5}{6}$ as old as his brother; what is the sum of their ages?

5. A man being asked his age replied that if to $\frac{1}{4}$ of his age 5 years be added, $\frac{1}{2}$ of $\frac{3}{5}$ of $\frac{5}{6}$ of the sum would be 5 years; how old was he?

6. What is the difference between 6 dozen dozen and half a dozen dozen?

7. What is the difference between $\frac{2}{3}$ of 15 and $\frac{2}{3}$ plus 15?

8. Which is the greater, and how much, $\frac{2}{3}$ or $\frac{3}{4}$? $\frac{4}{5}$ or $\frac{5}{6}$?

9. Mr. Booth spent $\frac{1}{3}$ of his money for a ticket to Chicago and return, $\frac{1}{6}$ of the remainder for board, $\frac{1}{9}$ of the whole sum for incidentals, and had left $\frac{4}{5}$ of \$45; how much had he at first?

10. Mr. Little gave $\frac{5}{7}$ of all the money he had for a horse; he then paid \$15 for a harness; if \$15 was $\frac{3}{8}$ of all the money he had left, how much had he at first?

11. A had $\frac{4}{7}$ of a ton of hay, which is $\frac{3}{4}$ of what B had; how much had B?

12. A has $\frac{4}{7}$ of a certain sum of money, which is $\frac{2}{3}$ of what B had; what fractional part has B more than A?

13. If $\frac{3}{4}$ of a yard of silk cost \$1$\frac{3}{5}$, how many yards can be bought for \$10$\frac{2}{3}$?

14. If 8 men can dig a ditch in 32 days, what part of it can 4 men dig in 9 days?

15. If 12$\frac{1}{2}$ tons of hay will feed 5 horses one year, how many tons will feed 12 horses 2 years?

16. If a man were 3 times as old as he is, $\frac{1}{3}$ of $\frac{1}{2}$ of $\frac{1}{4}$ of his age would be 3 years; how old is he?

17. If a number be diminished by $\frac{5}{7}$ of $\frac{4}{5}$ of itself, the remainder will be 63; what is the number?

18. If $\frac{5}{7}$ be added to a certain fraction, the sum will be $\frac{9}{10}$; what is the fraction?

19. A man is 50 years old, and $\frac{9}{25}$ of his age equals $\frac{3}{7}$ of the age of his wife; what is the sum of their ages?

20. If 6 pounds of coffee cost \$1$\frac{1}{3}$, what will 5$\frac{1}{3}$ pounds cost?

5.

1. Two boys, A and B, have \$35; $\frac{1}{2}$ of A's money is equal to $\frac{1}{5}$ of B's; how much money has each?

SOLUTION.—$\frac{2}{5}$, or A's money, will equal 2 times $\frac{1}{5}$, or $\frac{2}{5}$ of B's money. They will both have the sum of 1 (or $\frac{5}{5}$), and $\frac{2}{5}$, which equals $\frac{7}{5}$ times B's money. Therefore, \$35 is $\frac{7}{5}$ times B's money. \$35 is $\frac{7}{5}$ of 5 times $\frac{1}{7}$ of \$35. $\frac{1}{7}$ of \$35 is \$5, and 5 times \$5 are \$25, which equals B's money. \$35, the whole, less \$25, B's money, equals \$10, A's money. Therefore, B had \$25 and A had \$10.

2. Two men, C and D, have \$70. If $\frac{1}{3}$ of C's money equals $\frac{1}{5}$ of D's, how much money has each?

3. Two men have 200 sheep; if $\frac{1}{2}$ of the number that one has equals $\frac{1}{3}$ the number the other has, how many has each?

4. Two men own 360 acres of land. If $\frac{1}{4}$ of one man's share equals $\frac{1}{5}$ of the other man's share, how many acres has each?

5. If $\frac{2}{7}$ of a yard of velvet cost \$$2\frac{2}{3}$, how many yards can be bought for \$56?

6. How many pounds of honey at \$$\frac{3}{8}$ a pound can be bought for \$9?

7. If a pole 6 feet long casts a shadow $10\frac{1}{2}$ feet long, what shadow will a pole 8 feet long cast at the same time of day?

8. If 3 be added to both terms of the fraction $\frac{2}{3}$, will its value be increased or diminished, and how much?

9. If 3 be added to both terms of the fraction $\frac{3}{2}$, will its value be increased or diminished, and how much?

10. If $\frac{3}{4}$ be multiplied by $\frac{3}{4}$, will the value of the product be greater or less than each factor used? Why?

11. If $\frac{4}{3}$ be multiplied by $\frac{4}{3}$, will the value of the product be greater or less than each factor used? Why?

12. A bridle cost \$18, and $\frac{5}{9}$ of the cost of the bridle was $\frac{2}{7}$ of the cost of the saddle; what was the cost of the saddle?

13. A man sold his house for \$4000, which was $\frac{1}{3}$ more than it cost him; what did it cost him?

14. The difference between $\frac{5}{6}$ and $\frac{3}{8}$ of a number is 22; what is the number?

15. A man did $\frac{1}{4}$ of a piece of work the first day, $\frac{1}{5}$ of it the second day, and $\frac{1}{11}$ of the remainder the next day; what part of the work was left for him to do on the fourth day?

16. A farmer had his sheep in 3 fields; he had $\frac{1}{4}$ of them in one, $\frac{1}{3}$ of the remainder in another, and 60 sheep in the third; how many sheep did he own?

17. A woman is 36 years old, and $\frac{2}{9}$ of her age equals $\frac{1}{2}$ the age of her daughter; what is the sum of their ages?

6.

1. Alice has 12\frac{1}{2}$ and Sarah has $\frac{3}{5}$ as much; how much has Sarah?

2. How many souvenir spoons at 2\frac{1}{2}$ each can be bought for $\frac{5}{7}$ of $\frac{7}{9}$ of $45?

3. Divide 48 newspapers between 2 boys so that one will have 7 as often as the other has 9.

SOLUTION.—Since one has 7 as often as the other has 9, they will have 16 parts together; one will have 7 parts of the 16 parts, or $\frac{7}{16}$ of 48 papers, which equals 21 papers; the other will have 9 parts of the 16 parts, or $\frac{9}{16}$ of 48 papers, or 27 papers. Therefore, one will have 21 newspapers and the other 27 newspapers.

4. Divide $60 between two persons so that one will have $6 as often as the other has $9.

5. Divide 52 peaches between two persons so that one will have 5 peaches as often as the other has 8.

6. Divide $54 among 3 men so that one will have $2 as often as the second has $3 and the third has $4.

7. Divide 36 pineapples among 3 persons so that one has 5 as often as the second has 6 and the third 7 (or in the ratio of 5, 6 and 7).

8. Divide $68 between two persons in the ratio of $\frac{2}{3}$ and $\frac{3}{4}$.

EXPLANATION.—Fractions having similar denominators have the same ratio (or relation) as their numerators. Thus, $\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$. $\frac{8}{12}$ and $\frac{9}{12}$ have the same relation as 8 and 9. Hence, the problem may be read thus: Divide $68 between two persons in the ratio of 8 and 9.

9. Divide $70 among 4 persons in the ratio of 1, 2, 3 and 4.

10. George bought $\frac{2}{3}$ of a pound of candy; he gave his sister $\frac{1}{4}$ of it and his brother $\frac{1}{2}$ of the remainder; what part of a pound had he left?

11. A man bought a mackintosh and sold it for $\frac{7}{9}$ of its cost; how much did he lose?

12. When Mary is $\frac{3}{7}$ older than she now is, $\frac{1}{3}$ of her age will be 10 years; how old is she?

13. A man had $8\frac{3}{4}$ tons of hay and sold $5\frac{7}{8}$ tons of it; how much had he left?

14. The sum of two fractions is $\frac{5}{24}$; if one of them is $\frac{1}{8}$, what is the other?

15. The product of two numbers is 6; if one of them is 14, what is the other?

16. The product of two fractions is $\frac{8}{9}$; if one of them is $\frac{3}{4}$, what is the other?

17. How many bushels of potatoes can be bought for \$15 at $\$\frac{3}{4}$ a bushel?

18. If to $\frac{3}{4}$ of a man's age 18 years be added, the sum will be 6 times 9 years; how old is he?

19. A boy having $\$\frac{9}{10}$ gave $\$\frac{2}{5}$ for a knife and $\$\frac{1}{4}$ for a dinner; how much had he left?

20. If a yard of velvet cost $\$5\frac{3}{4}$, what will 12 yards cost?

21. If a mortgage of \$600 covers $\frac{5}{11}$ of the value of a house, what is the value of the house?

22. A man working 8 hours a day at $\$\frac{3}{10}$ an hour will earn how much in 6 days?

23. How much is received for 18 pails of berries, each containing $\frac{2}{3}$ of a peck, at $\$\frac{9}{10}$ a peck?

7.

1. If a clerk receive $\$18\frac{1}{2}$ a week and spend $\$8\frac{1}{2}$ a week how long will it take him to pay for a 20-acre farm at $\$30\frac{1}{2}$ an acre?

2. If $\frac{7}{8}$ of \$6400 is $\frac{2}{3}$ of my money, how much money have I?

3. There is a school in which $\frac{2}{5}$ of the boys study arithmetic, $\frac{1}{3}$ study grammar, $\frac{1}{9}$ study geography, and the rest, 15 boys, study history; how many boys in the school? how many in each subject?

4. A man sold a watch for \$64, which was $\frac{8}{9}$ of what it cost him; how much did he gain by the sale?

5. If 5 tons of hay will keep 4 horses through the winter; how many tons will keep 30 horses the same time?

6. If a cask of wine cost \$48, what will $\frac{3}{8}$ of $\frac{5}{3}$ of it cost?

7. A, B and C can build a boat in 5 days; A and B can build it in 8 days; how much of it can C do in one day? how long would it take C to build it alone?

8. If I line 8 yards of broadcloth $1\frac{1}{2}$ yards wide with lining $\frac{3}{4}$ of a yard wide, how many yards will line the whole?

9. If $1\frac{2}{3}$ yards of cloth cost \$3, what will $12\frac{1}{2}$ yards cost?

10. At $\$\frac{3}{5}$ a yard, how many yards of cloth can be bought for $\frac{15}{17}$ of \$51?

11. George and James together have \$60. James has $2\frac{1}{3}$ times as much as George; how much has each?

SOLUTION.—Since James has $2\frac{1}{3}$ times what George has, let 1, or $\frac{3}{3}$, represent what George has; then the two boys together will have $3\frac{1}{3}$ times what George has. The two together have \$60, which must be $3\frac{1}{3}$ times what George has. \$60 is $3\frac{1}{3}$, or $\frac{10}{3}$, of 3 times $\frac{1}{10}$ of \$60. $\frac{1}{10}$ of \$60 is \$6, and 3 times \$6 are \$18, what George has. \$60 — \$18 equals \$42, what James has. Therefore, George has \$18 and James has \$42.

12. A man gave $140 for a watch and chain, and the chain cost $\frac{2}{5}$ as much as the watch; find the cost of each?

13. Two girls together picked 45 quarts of berries; if one picked $\frac{4}{5}$ times as many as the other, how many did each pick?

14. A horse and wagon cost $270; the horse cost $1\frac{1}{4}$ times as much as the wagon; what did each cost?

15. Two men cut down 49 trees; if one cut $1\frac{1}{2}$ times as many as the other, how many did each cut?

16. A house and lot cost $4800; if the house cost $1\frac{2}{5}$ times as much as the lot, what did each cost?

17. Two boys worked 27 examples; if one worked $3\frac{1}{2}$ times as many as the other, how many did each work?

8.

1. A and B have 25 oranges; how many has each, if B has 5 more than A?

SOLUTION 1.—A's number plus 5 oranges equals B's number, and A's and B's number is twice A's plus 5, which, according to the conditions of the problem, equals 25 oranges. If twice A's plus 5 equals 25, twice A's equals 25 — 5, or 20. If twice A's equals 20, once A's equals $\frac{1}{2}$ of 20, or 10. Since B has 5 more than A, 10 plus 5, or 15, equals B's number. Therefore, A has 10 oranges and B, 15.

SOLUTION 2.—If A and B had had the same number, they would have had 25 oranges minus 5 oranges, or 20 oranges, and each would have had $\frac{1}{2}$ of 20 oranges, or 10 oranges. But since B had 5 more than A, he had 10 oranges plus 5 oranges, or 15 oranges. Therefore, etc.

2. John and Charles together had 39 cents; if John had 11 cents more than Charles, how many had each?

3. Divide the number 23 into two parts such that one shall be 5 less than the other.

4. The sum of two numbers is 87 and their difference is 9; what are the numbers?

5. Divide the number 49 into two parts (or numbers) which are to each other as $\frac{1}{3}$ to $\frac{1}{4}$.

SUGGESTION.—Fractions having a common denominator have the same relation to each other as their numerators.

SOLUTION.—$\frac{1}{3} = \frac{4}{12}$ and $\frac{1}{4} = \frac{3}{12}$. The problem becomes—divide the number 49 into two parts which are to each other as 4 to 3. 4 parts and 3 parts are 7 parts or 49; 4 of the 7 parts, or $\frac{4}{7}$ of 49, which equals 28, is one number, and $\frac{3}{7}$ of 49, or 21, is the other number. Therefore, the numbers are 28 and 21.

6. Divide the number 45 into two parts which are to each other as $\frac{1}{2}$ to $\frac{1}{3}$.

7. Divide the number 26 into three parts which shall be to each other as $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$.

8. Divide the number 77 into four parts which shall be to each other as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ and $\frac{1}{5}$.

9. If $\frac{2}{3}$ of the value of a house equals $\frac{4}{5}$ of the value of the lot, and both are worth $5500, what is the value of each?

10. A lady, being asked her age, replied that her daughter's age is 6 years, which is $\frac{4}{5}$ of $\frac{1}{4}$ of her age? what is the sum of their ages?

11. A man, being asked his age, replied: "If from $\frac{3}{5}$ of my age $\frac{1}{3}$ of it be subtracted, the remainder will be 24 years." What is his age?

12. A watch and chain cost $150, and $\frac{2}{3}$ of the cost of the watch, plus $15, equals $95; what was the cost of each?

13. Martha gave $\frac{3}{4}$ of her money to the poor and then found $\frac{2}{3}$ as much as she gave away; if she then had $30, how much had she at first?

14. A watch cost $40, and this is $\frac{8}{7}$ of $\frac{7}{8}$ of the cost of the watch and chain together; how much did the chain cost?

15. If 10 oxen eat 4 acres of grass in 6 days, in how many days will 30 oxen eat 6 acres?

SUGGESTION.—Reason to *unity* first.

9.

1. B gave $\frac{5}{7}$ of all his money for a cow; he paid $12 for hens, which was $\frac{3}{4}$ of all the money he had left; how much money had he at first?

2. Thomas, after spending $\frac{2}{5}$ of his money, found that $16 was $\frac{2}{3}$ of what he had left; how much money had he at first?

3. $\frac{1}{3}$ of the length of a pole is in the air, $\frac{1}{4}$ in the water, and 10 feet in the ground; what is the height of the pole?

4. Peter gave 10 cents for a pie which is $\frac{3}{5}$ of $\frac{5}{9}$ of what he gave for a dinner; what was the cost of his dinner?

5. A machinist whose wages were 4\frac{1}{2}$ a day, by overwork made $\$\frac{3}{5}$ a day more; how much did he earn in 10 days?

6. If a man spend 3\frac{5}{8}$ for fuel and 11\frac{1}{3}$ for board from a salary of $30 a week, how much will remain?

7. What number multiplied by $1\frac{5}{9}$ will produce 56?

8. A boy spent $\frac{4}{7}$ of his money and the remainder he gave for 9 sheep at 2\frac{1}{3}$ each; how much money had he at first?

9. A certain coin is made of 22 parts copper and 4 parts alloy; what fractional unit does the copper represent? the alloy?

10. A bookkeeper spends $800 a year; if this sum is $\frac{5}{9}$ of his salary, what is his salary?

DECIMAL FRACTIONS.

1. If a unit is divided into 10 equal parts, and each of these parts into 10 other equal parts, and so on indefinitely, the fractions thus found are called *decimal fractions.*

2. A DECIMAL FRACTION is a fraction having for its denominator 10, 100, 1000, etc. Or, it is one of the *decimal* divisions of a unit.

3. Decimal Fractions, commonly called *decimals,* are usually expressed by writing the numerator only with a point before it, called the decimal point. Thus,

$\frac{2}{10}$	is written	.2	and read	2 tenths.
$\frac{5}{10}$	is written	.5	and read	5 tenths.
$\frac{7}{100}$	is written	.07	and read	7 hundredths.
$\frac{15}{1000}$	is written	.015	and read	15 thousandths.
$\frac{25}{10000}$	is written	.0025	and read	25 ten-thousandths.
$\frac{125}{100}$	is written	1.25	and read	125 hundredths.

4. The decimal point is also called a *separatrix* when it is used to separate a whole number and a decimal.

Thus, 34.56.

5. Decimal Fractions arise from dividing the *unit,* first into 10 equal parts, and then each of these parts into 10 equal parts, and so on.

Thus, $1 \div 10$, or $\frac{1}{10}$ of 1, $= \frac{1}{10} = .1$; $\frac{1}{10} \div 10$, or $\frac{1}{10}$ of $\frac{1}{10}$, $= \frac{1}{100} = .01$; $\frac{1}{100} \div 10$, or $\frac{1}{10}$ of $\frac{1}{100} = \frac{1}{1000} = .001$.

6. It will be seen from the above examples that there must be as many places in the decimal as there are ciphers in the denominator of the fraction; and that every decimal has for its denominator 1, with as many ciphers annexed as there are places in the given decimal. When the numerator does not contain as many places as

there are ciphers in the denominator, prefix ciphers to the numerator until the number of places is equal to the number of ciphers in the denominator, and prefix the decimal point.

Thus, $\frac{7}{100} = .07$; $\frac{25}{10000} = .0025$, etc.

7. The relation of decimals to whole numbers and to each other, and also the names of their different orders and places, may be shown by the following

TABLE.

NAMES.	Hund-millions.	Ten-millions.	Millions.	Hund-thousands.	Ten-thousands.	Thousands.	Hundreds.	Tens.	UNITS.	.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hund-thousandths.	Millionths.	Ten-millionths.	Hund-millionths.
ORDERS.	9th	8th	7th	6th	5th	4th	3d	2d	1st		2d	3d	4th	5th	6th	7th	8th	9th
UNITS.	3	3	3	3	3	3	3	3	3	.	3	3	3	3	3	3	3	3
	INTEGERS.										DECIMALS.							

8. The number is read, 333 million 333 thousand 333, and 33 million 333 thousand 333 hundred-millionths.

9. A decimal takes the *name* of its right-hand order. Thus, in the number 234.86275, the decimal takes the name of the right-hand order, or 5, which is *hundred-thousandths,* and the number is read 234 and 86275 hundred-thousandths.

10. The unit 1 is the base or starting point of notation and numeration of both whole numbers and decimals, and for convenience the decimal point is placed at the *right* of units instead of under it, and may be supposed to occupy units place.

11. In writing decimals, as in writing integers, all vacant orders must be filled with ciphers.

12. To read decimals requires two numerations; first, *from* units, to find the name of the denominator; and second, *toward* units, to find the value of the numerator.

Read the following—

.5	.12	.007	.003	.365	.88	.4567
.15	.01	.33	.3	.375	.675	.0067
.06	.125	.04	.30	.08	.6755	.0007

13. A MIXED NUMBER is an integer and a decimal written together.

14. The word *and* is always read between the integer and the decimal.

Thus, 7.85 is read seven and eighty-five hundredths.

15. Read the following: .37, .02, 2.5, 37.5, 35.05, 526.007.

TO REDUCE A DECIMAL TO A FRACTION.

1. Reduce .42 to a common fraction.

SOLUTION.—The denominator of a decimal is 1, with as many ciphers annexed as there are figures in the numerator; therefore, the denominator of 42 is 100, or .42 = $\frac{42}{100}$.

Reduce the following decimals to common fractions and to their lowest terms:

2. .5, .04, .75, .62½, or .625, .37½, .05, .50, .875.

3. Read the decimals: .3 : .30 : .300.

4. To what common fraction is each of the above decimals equivalent?

5. How does $\frac{3}{10}$ compare in value with $\frac{30}{100}$?

How does annexing a cipher to a decimal affect its value?

Ans. Annexing a cipher to a decimal does not alter its

value, because denominator as well as numerator is multiplied by 10.

1. Reduce $\frac{3}{8}$ to a decimal fraction.

SOLUTION.—A fraction indicates division, and its value is the numerator divided by the denominator. $\frac{3}{8}$ equals $\frac{1}{8}$ of 3. 3 equals 30 tenths (3.0); $\frac{1}{8}$ of 30 tenths is 3 tenths and 6 tenths remaining; 6 tenths equals 60 hundredths (.60); $\frac{1}{8}$ of 60 hundredths is 7 hundredths and 4 hundredths remaining; 4 hundredths equals 40 thousandths (.040); $\frac{1}{8}$ of 40 thousandths equals 5 thousandths. Therefore, $\frac{3}{8} = \frac{3.000}{8} = .375$.

Point off as many decimal places from the right, in the result, as there are ciphers annexed to the numerator.

Reduce the following to the form of decimals:

2. $\frac{1}{2}$. *4.* $\frac{1}{5}$. *6.* $\frac{2}{5}$. *8.* $\frac{4}{5}$. *10.* $\frac{3}{12}$. *12.* $\frac{5}{8}$. *14.* $4\frac{1}{4}$.

3. $\frac{1}{4}$. *5.* $\frac{1}{8}$. *7.* $\frac{3}{4}$. *9.* $\frac{3}{15}$. *11.* $\frac{7}{8}$. *13.* $2\frac{1}{2}$. *15.* $12\frac{1}{8}$.

ADDITION OF DECIMALS.

1. Add .3, .02, $2\frac{1}{2}$ (which equals 2.5), $16\frac{3}{4}$ and 5.4.

SOLUTION.—We write the numbers with units of the same order in the same column; that is, units under units, tenths under tenths, etc. Add them as whole numbers and place the decimal point of the sum directly under those of the numbers added.

Thus, in the above example—

.3	As annexing a cipher to a decimal does not alter its	
.02	value, the numbers may be expressed as follows.	.30
2.5	This also reduces the decimals to a common	.02
16.75	denominator.	2.50
5.4		16.75
24.97		5.40
		24.97

2. Add 7, 3.25, 5.1.

3. Add 4.25, 6.03, 6.6.

4. Add \$6.30, \$4.01, \$3.15.

5. 2.25, 3.35, 4.45.

6. 7.7, 8.8, 9.9.

7. \$12.50, \$7.09, \$5.10.

SUBTRACTION OF DECIMALS.

1. From 42.25 take 17.375.

SOLUTION.—We write the subtrahend under the minuend, placing units under units, tenths under tenths, etc. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points in the minuend and subtrahend.

Thus, in the above example—

$$\begin{array}{r} 42.250 \\ \underline{14.375} \\ 27.875 \end{array}$$

2. From 18.75 take 8.65.
3. From 20.45 take 7.325.
4. From 16.125 take 12.375.
5. From \$7.50 take \$4.12½.
6. Take .85 from 1.25.
7. Take 9.9 from 99.
8. Take 1.01 from 10.1.
9. Take \$2.42 from \$5.13.

MULTIPLICATION OF DECIMALS.

1. Multiply .25 by .5.

$.25 = \frac{25}{100}$

$.5 = \frac{5}{10}$

$\frac{25}{100} \times \frac{5}{10} = \frac{125}{1000} = .125$

Therefore, $\begin{array}{r} .25 \\ \underline{.5} \\ .125 \end{array}$

EXPLANATION. — Since hundredths multiplied by tenths produce thousandths, point off three decimal places; or, multiply as in whole numbers, and from the right of the product point off as many figures for decimals as there are decimal places in both factors.

2. Multiply .04 by 6.
3. Multiply .25 by .7.
4. Multiply 7.5 by 4.
5. Multiply \$4.25 by 3.
6. Multiply 5 by .5.
7. Multiply .5 by .5.
8. Multiply 9.25 by 10.
9. Multiply \$8.40 by 7.
10. Multiply 2.2 by .2.
11. Multiply .22 by .2.
12. Multiply .45 by .06.
13. Multiply .50 by .05.
14. Multiply 3.5 by .7.
15. Multiply 234 by .01.
16. Multiply 5.05 by 5.
17. Multiply 61.3 by .4.

DIVISION OF DECIMALS.

1. Divide .125 by .5.

$$.125 = \frac{125}{1000}$$

$$.5 = \frac{5}{10}$$

$$\frac{125}{1000} \div \frac{5}{10} = \frac{\overset{25}{\cancel{125}}}{\underset{100}{\cancel{1000}}} \times \frac{\cancel{10}}{\cancel{5}} = \frac{25}{100}$$

Therefore, $.5\,\underline{)\,.125}$

$.25$

EXPLANATION.—We divide as in whole numbers, and from the right of the quotient point off as many places for decimals as the decimal places in the dividend exceed those in the divisor.

Or, since the divisor and quotient are the two factors, which, being multiplied together, produce the dividend, we point off two decimal figures in the quotient to make the number in the two factors equal to the product or dividend.

Or, since $\frac{125}{1000}$ is the product of the divisor and quotient, dividing the numerator of the dividend by the numerator of the divisor, we have 25 for the numerator of the quotient; dividing the denominator of the dividend by the denominator of the divisor, we have 100 for the denominator of the quotient, and the entire quotient, $\frac{25}{100}$, which equals .25.

NOTE 1.—If the number of figures in the quotient be less than the excess of the decimal places in the dividend over those in the divisor, the deficiency must be supplied by prefixing ciphers.

NOTE 2.—The dividend must always contain at least as many decimal places as the divisor before commencing the division.

2. Divide .35 by 5.
3. Divide 7.5 by 2.5.
4. Divide 10.5 by 2.1.
5. Divide .125 by .25.
6. Divide 17.28 by .12.
7. Divide 1.728 by 1.2.
8. Divide 172.8 by .12.
9. Divide .1728 by .12.

PROBLEMS.

1. If a pound of tea costs \$.60 and a pound of coffee \$.38, what is the cost of both?

SOLUTION.—They will both cost the sum of \$.60 and \$.38, which is \$.98. Therefore, they will both cost \$.98.

2. If Benjamin earned $.75 and spent $.28, how much had he left?

3. A man walked 30.7 miles the first day and 40.4 miles the second day; how far did he walk in the two days? How much farther did he walk the second day than the first?

4. A man paid two bills; the first was $7.63 and the second was the balance of a twenty-dollar bill after the first payment. How much was his second bill?

5. A lady bought shoes for $3.75 and a hat for $7.25. How much less than $19.50 did she spend?

6. The sum of two numbers is 25; one of the numbers is 6.24; what is the other number?

7. How many rods will enclose a field, the four sides of which are respectively 16 rods, 14.5 rods, 8.25 rods and 15 rods?

8. If 5 boys receive $7.50, how much does each receive?

9. At 12 cents each, how many books can be bought for $17.28?

10. If Ned paid $9.25 for one coat, what did he pay for 10 coats at the same rate?

11. Jane bought 6 pencils at $.04 each, and 7 handkerchiefs at $.25 each; how much money did she spend?

12. How much change had she left from $5?

13. What is the difference in cost between 7 caps at $1.25 each and 5 caps at $1.50 each?

14. The average amount of rain at Cincinnati in the summer months is 13.7 inches and in the winter months it is 11.15 inches; what is the difference?

15. One man owned $.37\frac{1}{2}$ of an iron foundry and another owned $.16\frac{1}{4}$; what decimal part did both own?

16. If 21 pears cost $1.05, what will one pear cost?

17. What is the cost of $1\frac{1}{2}$ tons of hay at $17.50 a ton?

18. If 35 men share equally .245 of a gold mine, what decimal part of the mine does each own?

19. A man owns $\frac{7}{8}$ of an iron foundry; he sells $\frac{4}{21}$ of his share; what decimal will express the part of the whole that he sold?

20. A man sold .25 of his farm at one time, $.12\frac{1}{2}$ at another; what decimal part of his farm has he left?

21. How many umbrellas at $1.50 each can be bought for $10.50?

22. A lady bought 4 handkerchiefs at $\$.62\frac{1}{2}$ apiece, 3 handkerchiefs at $1.50 each, and one handkerchief for $1.25; how much did she spend for handkerchiefs?

23. Which is the greater, and how much, the product of 2.5 multiplied by 10, or the quotient of 25 divided by .1?

24. How many coats can be made from 32.4 yards of cloth, allowing 2.7 yards for each coat?

25. When railroad fare is 3 cents a mile, how far can you ride for $2.88?

26. At .6 of a dollar per yard, what will 1.2 yards of cloth cost?

27. When tenths are multiplied by tenths, what denomination is the product? Tenths by hundredths? Hundredths by hundredths? Dividing thousandths by tenths? Tenths by thousandths?

28. If 33 gallons of oil cost $36.30, what is the cost per gallon?

29. From 4 take 3.25, add .25, subtract .5, divide by .5 and multiply the result by .1. What is the answer?

30. A boy who earned $7.75 a week spent $2.30 of it every week; how much did he save in 6 weeks?

DENOMINATE NUMBERS.

1. A DENOMINATE NUMBER is one whose unit is named.

Thus, 18 inches, and 2 feet 4 inches are denominate numbers.

2. REDUCTION OF DENOMINATE NUMBERS is the process of changing the *denomination* of a number without changing its *value.*

Thus, 2 yards may be expressed, 6 feet or 72 inches; 2 gallons, as 8 quarts or 32 pints.

TABLES OF MONEY.

U. S. MONEY.

1. UNITED STATES or FEDERAL MONEY is the legal currency of the United States.

10 mills (m.)	make	1 cent.....ct. or ¢.
10 cents	"	1 dime.....d.
10 dimes	"	1 dollar....$.
10 dollars	"	1 eagle.....E.

2. In writing dollars and cents, separate the dollars and cents by a period. If the number of cents is less than 10, place a cipher after the period.

Thus, $5.15 is read 5 dollars, 15 cents, $6.03 is read 6 dollars, 3 cents; $10.03½ is read 10 dollars, 3½ cents.

ENGLISH MONEY.

3. ENGLISH MONEY is the national currency of Great Britain.

4 farthings (far.)	make	1 penny.....d.
12 pence	"	1 shilling....s., or sh.
20 shillings	"	1 pound (£)..or sovereign, sov.

1. How many cents in 5 dimes? in 4 dollars? in 3 eagles?

SOLUTION.—Since in 1 dime there are 10 cents, in 5 dimes there are 5 times 10 cents, which are 50 cents. Therefore, etc.

2. How many dollars in 9 eagles? in 40 dimes? in 500 cents?

3. How many eagles in 50 dollars? in 300 dimes? in 6000 cents?

4. What part of \$1 is $12\frac{1}{2}$¢? $16\frac{2}{3}$¢? 10¢? 20¢? 25¢?

5. How many farthings in 4 pence? in 7 pence?

6. How many pence in 8 farthings? in 6 farthings?

7. How many pence in 2 shillings? in 6 shillings?

8. How many shillings in 24 pence? in 48 pence?

9. What part of two pence is 6 farthings, and what part of 4 shillings is 6 pence?

SOLUTION.—2 pence equals 8 farthings, and since 6 is $\frac{6}{8}$ or $\frac{3}{4}$ of 8, 6 farthings must be $\frac{3}{4}$ of 8 farthings, or of 2 pence. Therefore, etc.

10. What part of 16 pence is $\frac{2}{3}$ of a shilling?

11. What part of 24 shillings is $\frac{3}{5}$ of a pound?

12. What part of 36 shillings is $\frac{9}{10}$ of a pound?

TABLES OF WEIGHT.

TROY WEIGHT.

1. TROY WEIGHT is used in weighing gold, silver and precious stones.

24 grains (gr.)	make	1 pennyweight	pwt.
20 pennyweights	"	1 ounce	oz.
12 ounces	"	1 pound	lb.

2. How many ounces in 5 pounds? in 6 lb.? in 7 lb.?

SOLUTION.—Since in one pound there are 12 ounces, in 5 pounds there are 5 times 12 ounces, or 60 ounces. Therefore, etc.

3. How many pennyweights in 48 grs.? in 72 grains?

4. What is the value of 3 ounces of gold if 3 pwt. are worth $3?

Apothecaries' Weight.

2. Apothecaries' Weight is used in prescribing and mixing dry medicines.

20 grains (gr.)	make	1 scruple ..sc.
3 scruples	"	1 dram....dr.
8 drams	"	1 ounce ...oz.
12 ounces	"	1 pound...℔.

Apothecaries' Fluid Measure

60 minims (m.)	make	1 fluid drachm.
8 fluid drachms	"	1 fluid ounce.
16 fluid ounces	"	1 pint.
8 pints	"	1 gallon

The above table is used in retailing liquid medicines.

1. How many grains in 3 sc.? in 5 sc.? in 2 oz.? in 1 lb.?

2. How many scruples in 40 grains? in 2 oz. and 3 drs.?

3. If 5 grains of medicine cost 10 cents, what will 2 drams and 3 scruples cost?

4. If 6 fluid drachms cost 12 cents, what will one ounce cost?

Avoirdupois Weight.

3. Avoirdupois Weight is used for all the ordinary purposes of weighing.

16 ounces (oz.)	make	1 pound.....℔.
25 pounds	"	1 quarter.....qr.
4 quarters or 100 pounds	"	1 hundredweight..cwt.
20 hundredweight	"	1 ton .....T.

1. How many quarters in 500 lbs.? in 75 lbs.? in 100 lbs.?

2. How many hundredweight in 40 qr.? in 300 lbs.?

3. What will 5 pounds of starch cost if 5 ounces cost $3\frac{3}{4}$ cents?

4. Which is the heavier, a pound of gold or a pound of lead? an ounce of silver or an ounce of feathers?

TABLES OF MEASURE.

Long Measure.

1. Long Measure is used in measuring lengths and distances.

12 inches (in.)	make	1 foot.....ft.
3 feet	"	1 yard....yd.
$5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet	"	1 rod.....rd.
320 rods	"	1 mile....mi.
3 miles	"	1 league ..lea.

1. How many inches in 3 feet? in 5 ft.? in 6 ft. 4 in.?

2. How many feet in 48 inches? in 60 in.? in 72 in.?

3. How many yards in 45 feet? in 66 ft.? in 72 in.?

4. How many miles in 32 leagues? in 75 leagues?

5. What part of 3 yards is 2 feet?

Square Measure.

2. Square Measure is used in computing areas and surfaces.

144 square inches (sq. in.)	make	1 square foot...sq. ft.
9 square feet	"	1 square yard...sq. yd.
$30\frac{1}{4}$ square yards	"	1 square rod....sq. rd.
160 sq. rds.	"	1 acre .....A.
640 acres	"	1 square mile...sq. mi.

1. How many square feet in 78 square yards?

2. How many square yards in 36 sq. ft.? in 63 sq. ft.?

3. What part of an acre is 80 sq. rds.? 120 sq. rds.?

4. What is the difference between 3 feet square and 3 square feet?

5. What is the difference between 4 inches square and 4 square inches?

Cubic Measure.

3. Cubic Measure is used in finding solid contents or volumes.

1728 cubic inches (cu. in.)	make	1 cubic foot...cu. ft.
27 cubic feet	"	1 cubic yard ..cu. yd.
16 cubic feet	make	1 cord foot.....cd. ft.
8 cord feet or 128 cu. ft.	"	1 cord of wood..cd.

$24\frac{3}{4}$ cubic feet make 1 perch of stone or masonry.

In *practice,* masons estimate $16\frac{1}{2}$ cubic feet 1 perch of stone or masonry. $16\frac{1}{2}$ ft. long, 1 ft. high and 1 ft. thick.

1. How many cords in 48 cord feet? in 256 cubic feet?

2. How many cubic feet in 3 cords?

3. How many cubic feet in 3 cubic yards?

MEASURES OF CAPACITY.

Table for Liquid Measure.

1. Liquid Measure is used in measuring milk, wine, vinegar, etc., and is often called *Wine Measure.*

4 gills (gi.)	make	1 pint....pt.
2 pints	"	1 quart...qt.
4 quarts	"	1 gallon ..gal.

The standard barrel contains $31\frac{1}{2}$ gallons.

2 barrels or 63 gallons make 1 hogshead (hhd.)

1. How many gills in 6 pints? in 3 quarts? in 2 gallons?

2. How many pints in 24 gills? in 7 quarts? in 3 gallons?

3. How many quarts in 40 gills? in 12 pints? in 7 gallons?

4. How much ale at 50 cents a gallon must be given for 8 quarts of wine at $2 a pint?

Table for Dry Measure.

2. Dry Measure is used in weighing dry substances, as salt, fruit, grain, etc.

2 pints (pt.)	make	1 quart ....qt.
8 quarts	"	1 peck.....pk.
4 pecks or 32 qts.	"	1 bushel...bu.

1. How many pints in 4 quarts? in 3 pecks? in 3 bushels?

2. How many quarts in 8 pecks? in 4 bushels? in 16 pints?

3. How many pecks in 48 pints? in 48 quarts?

4. How many bushels in 64 quarts? in 128 pints?

5. At 20 cents a peck, how many bushels of apples can be bought for $8?

Solution.—Since 1 peck costs 20¢, 4 pecks or one bushel will cost 4 times 20¢, or 80¢. As many bushels can be bought for $8 as 80¢ is contained times in $8, which is 10 times. Therefore, etc.

6. What is the difference in cost between 5 bushels 3 quarts of salt at 4 cents a quart and 10 bushels 3 pecks of apples at 50 cents a bushel?

7. How many pecks must be added to 3 pecks to make 4 bushels?

MEASURES OF TIME AND CIRCLES.

TABLE OF TIME.

1. TIME is a measured portion of duration.

60 seconds (sec.)	make	1 minute	min.
60 minutes	"	1 hour	hr.
24 hours	"	1 day	da.
7 days	"	1 week	wk.
4 weeks	"	1 month	mo.
12 months	"	1 year	yr.
365 days	"	1 common year	c. yr.
366 days	"	1 leap year	l. yr.
100 years	"	1 century	cen.

In most business transactions, 30 days are reckoned as one month.

2. The following will be of assistance in remembering the number of days in each month:

> "Thirty days hath *September*,
> *April*, *June* and *November*;
> All the rest have thirty-one
> Excepting February alone,
> To which we twenty-eight assign,
> Till leap-year gives it twenty-nine."

3. Every year exactly divisible by 4, centennial years excepted, is a leap year. 1893 is a common year. 1896 is a leap year.

4. Every *centennial* year exactly divisible by 400 is a leap year; the rest are common years. 2000 is a leap year. 1900, common year.

Although divisible by 4, 1900 is a common year because not divisible by 400.

1. How many seconds in 2 min.? in 3 min.? in 6 min.?

2. How many minutes in 3 hours? in 120 seconds?

3. How many hours in 4 days? in 240 minutes?

4. How many days in 7 weeks? in 56 hours?

5. Name the months which have 30 days each. Those having 31 days. Repeat the stanza.

Circular Measure.

2. Circular Measure is used to measure arcs of circles, angles, etc.

60 seconds (″)	make	1 minute	′
60 minutes	"	1 degree	°
30 degrees	"	1 sign	S.
12 signs	"	1 circumference	C.

Note.—90° = 1 quadrant or right angle. 60° = 1 sextant.

1. How many minutes in 180″? in 240″?

2. How many seconds in 3′? in 4′? in 5′?

3. How many signs in 90°? in 180°? in 360°?

Table for Counting.

12 units (u.)	make	1 dozen	doz.
12 dozen	"	1 gross	gr.
12 gross	"	1 great gross	g. gr.
20 units	"	1 score.	

1. How many eggs in seven dozen? units in 1 gross?

2. How many years in 4 score? in 3 score and 10 years?

3. How many dozen in 144? in a great gross?

Table for Paper.

24 sheets (sh.)	make	1 quire	qr.
20 quires	"	1 ream	R.
2 reams	"	1 bundle	bu.
5 bundles	"	1 bale	B.

Note.—480 sheets make 1 ream.

1. How many sheets in 6 quires? in 9 quires?
2. How many quires in 72 sheets? in 144 sheets?
3. How many sheets in 4 reams? in a half ream?
4. How many reams in 4 bundles? in 1 bale?

Miscellaneous Tables.

231 cubic inches	make	1 gallon.
2150.4 cubic inches	"	1 bushel.
24 hours	"	360 degrees.
5760 grains	"	1 pound Troy.
5760 grains	"	1 pound Apothecaries'.
7000 grains	"	1 pound Avoirdupois.
63360 inches	"	1 statute mile.

In most states—

32	pounds make	1	bushel	of oats.
48	"	"	"	barley.
56	"	"	"	corn or rye.
60	"	"	"	wheat.
60	"	"	"	potatoes.
196	"	"	barrel	of flour.

Problems.

1. How many bushels in 64 pounds of oats?
2. How many bushels in 180 pounds of wheat?
3. How many bushels in 224 pounds of corn?
4. How many bushels in 240 pounds of potatoes?
5. How many pounds in 3 barrels of flour?
6. How many pounds in 2 bushels of wheat?
7. How many pounds in 3 bushels of oats?
8. How many pounds in 3 bushels of corn or rye?
9. How many pounds in 3 bushels of barley?

Examples Illustrating the Use of the Tables.

1. What will 6 books cost at 6 dimes 5 cents each?

2. How many cents in 3 dollars, 4 dimes, 6 cents?

3. How many pence in 30 shillings?

4. How many farthings in 1 sovereign?

5. How many pounds in 480 shillings?

6. How many grains in 5 pennyweight?

7. What part of a pound Troy are 4 ounces?

8. How many parts pure gold in a ring 18 carats fine? (24 carats in pure gold.)

9. How many doses of 8 grains each can be made from an ounce of quinine?

10. How many pounds in 2 tons, 10 cwt., 25 lbs.?

11. How many tons in 2200 cwt.?

12. How many ounces in 7 pounds of sugar?

13. How many feet in 72 inches? in 14 yards?

14. What will 9 yards of ribbon cost at 1 cent an inch?

15. How many feet in 1 mile?

16. How many inches in 1 mile?

17. Reduce 13 feet 8 inches to inches.

18. How many square rods in 1 acre?

19. How many acres in 1 square mile?

20. How many square inches in 1 square foot?

21. How many cubic feet in 1 cubic yard?

22. If a pile of wood contains 768 cubic feet, how many cords does it contain?

23. How many pints in 11 quarts?

24. What is the cost of 10 gallons of milk at 5¢ a quart?

25. What will 3 bushels of potatoes cost at 20¢ a peck?

26. How many pints in 1 bushel?

27. How many minutes in 3 days?

28. How many days in a common year?

29. In 3 centuries, how many years?

30. How many days in the three summer months, June, July, August?

31. In the autumn months, September, October and November?

32. In the winter months, December, January and February?

33. In the spring months, March, April and May?

34. Which are the leap years?

35. How many degrees in a circumference?

36. How many degrees in a quadrant?

37. How many minutes in 3 degrees?

38. How old is a man who is 3 score and 10?

39. How many eggs in 6½ dozen?

40. How many sheets in 4 quires of paper?

41. How many sheets in 2 reams?

42. What will 2 bu. of oats cost at 1½ cents a pound?

43. How many pounds in 9 bushels of potatoes?

44. How many pounds in 5 barrels of flour?

45. How many pounds in 3 bushels of wheat?

46. How many pounds in 5 bushels of barley?

47. Which of the following are leap years: 1700? 1760? 1776? 1800? 1876? 1890? 1900? 2000?

48. My watch ticks 4 times in a second; how many times will it tick in 10 hours?

49. If I sell 1½ bushels of oats at 1½ cents a pound, and 1½ dozen of eggs at 1½ cents for each egg, how much money do I receive?

50. What is the difference in value between a barrel of flour at 3¢ a pound and 5 bushels of potatoes at the rate of 3 pounds for 2 cents?

PERCENTAGE.

1. One per cent. of a number is one-hundredth of it; two per cent. is two-hundredths of it; and any per cent. of a number is so many hundredths of it.

2. How many hundredths of a number is 2 per cent. of it? 3 per cent. of it? 4 per cent. of it? 5 per cent. of it? 20 per cent. of it?

3. What is $\frac{1}{100}$ of 100 lbs.? $\frac{2}{100}$ of \$100? $\frac{3}{100}$ of 100 books?

4. What part of 100 lbs. is 1 lb.? 4 lbs.? 10 lbs.?

5. PERCENTAGE is the process of computing by *hundredths.*

6. PER CENT. is derived from the Latin words *per centum,* and means *by* or *on the hundred;* and any per cent. of a number or quantity is so many hundredths of that number or quantity.

Thus, 5 per cent. of a quantity means 5 of every 100 or $\frac{5}{100}$ of the quantity; that is, \$5 of every \$100; 5 cents of every 100 cents; 5 apples of every 100 apples, etc.

7. The SIGN OF PER CENT. is %. It is read per cent.

Thus, 4% is read 4 per cent. $\frac{1}{2}$%, $\frac{1}{2}$ per cent. $4\frac{1}{2}$%, $4\frac{1}{2}$ per cent.

8. How many hundredths of a number is 3 per cent. of it? 9 per cent. of it? 4%? 10%? 15%? 25%? 75%?

9. What *per cent.* of a number is $\frac{2}{100}$ of it? $\frac{5}{100}$ of it? $\frac{8\frac{1}{2}}{100}$ of it? $\frac{12\frac{1}{2}}{100}$ of it?

10. What per cent. of a number is $\frac{1}{2}$ of it?

SOLUTION.—Since the whole of any number or thing is $\frac{100}{100}$, $\frac{1}{2}$ of the same is $\frac{1}{2}$ of $\frac{100}{100}$ or $\frac{50}{100}$, equal to 50%. Therefore, $\frac{1}{2}$ of a number is 50 per cent. of it.

11. What per cent. of a number is $\frac{1}{5}$ of it? $\frac{1}{8}$? $\frac{1}{10}$? $\frac{3}{4}$? $\frac{4}{5}$?

12. *Per cent.* may also be expressed as follows:

By the sign %, by a decimal fraction and by a common fraction.

Thus—

Per Cent.	Decimal Fr.	Common Fr.	Per Cent.	Decimal Fr.	Common Fr.
1% =	.01	$= \frac{1}{100}$.	25% =	.25 $(\frac{25}{100})$	$= \frac{1}{4}$.
3% =	.03	$= \frac{3}{100}$.	$2\frac{1}{2}$% =	.025 $(\frac{25}{1000})$	$= \frac{1}{40}$.
4% =	.04 $(\frac{4}{100})$	$= \frac{1}{25}$.	$\frac{1}{2}$% =	.005 $(\frac{5}{1000})$	$= \frac{1}{200}$.
5% =	.05 $(\frac{5}{100})$	$= \frac{1}{20}$.	$\frac{3}{4}$% =	.0075 $(\frac{75}{10000})$	$= \frac{3}{400}$.
6% =	.06 $\frac{6}{100}$	$= \frac{3}{50}$.	125% =	1.25 $(\frac{125}{100})$	$= 1\frac{1}{4}$.

13. The decimal point must not be prefixed when the sign % or the words per cent. are used. 20 % is not the same as .20%.

20% $= \frac{20}{100}$ or $\frac{1}{5}$. .20% $= \frac{20}{100}$ of $\frac{1}{100}$ or $\frac{1}{500}$.

14. The quantities considered in Percentage are—

1. THE BASE, the number on which the percentage is computed.

2. THE RATE, the number of hundredths of the base.

3. THE PERCENTAGE, the result obtained by taking a certain per cent. of the base.

4. THE AMOUNT or DIFFERENCE, the number obtained by adding the percentage to or subtracting it from the base.

TO FIND THE PERCENTAGE.

1. What is 5% of $60?

SOLUTION 1.—5% of anything is $\frac{5}{100}$ or $\frac{1}{20}$ of it; and $\frac{1}{20}$ of $60 is $3. Therefore, 5% of $60 is $3.

SOLUTION 2.—5% of anything is .05 of it, and .05 of $60 is $3. Therefore, etc.

2. What is 10% of $250? 20% of 300 miles?

3. What is 25% of 60 apples? 2% of 100 bu.?

4. What is 75% of 16 dollars? $12\frac{1}{2}$% of 80 lbs.?

5. What is 15% of 60 yards? $16\frac{2}{3}$% of 48?

6. A clerk receiving $50 a month had his salary raised 40%. What did he then receive?

7. An electric car running 12 miles an hour increases its speed 25%. How far does it then run in an hour?

8. If a man who had 200 pounds of wool sold 40% of it, how much had he left?

9. A house and lot purchased for $4000 increased 25% of its value; how much was it then worth?

10. A man having an income of $3000 spent $33\frac{1}{3}$% of it; how much had he left?

11. What is the difference between .05 of $2000 and .05% of the same amount?

12. A man owning a foundry worth $12000 sold 20% of it; what was the value of the part that he sold?

13. Mr. Todd, owing $4000, was able to pay only 40% of it; how much did he pay?

14. James sold his watch for $200; he sold the chain for 15% of the value of the watch; what did he receive for both?

15. My income last year was $600; this year it has been increased 20%; what is my income at present?

16. Of a salary of $2000, Mr. Walters paid 20% for board, 15% for books, 5% in traveling, 10% for other expenses; how much did he spend for each item? how much did he spend in all? how much did he save?

17. Mr. Blank owes $700. He paid at one time 10% of it, at another 10% of the remainder; how much does he still owe?

To Find the Rate Per Cent.

1. What per cent. of 50 is 5?

Suggestion.—The above is equivalent to—What part of 50 is 5?

Solution.—5 is $\frac{5}{50}$ or $\frac{1}{10}$ of 50; and $\frac{1}{10}$ of 100% is .10 or 10%. Therefore, 5 is 10% of 50.

2. What per cent. of $300 is $12? $15? $30? $60?

3. What per cent. of 120 is 20? 40? 60? 80? 90?

4. What per cent. of 80 bu. is 12 bu.? 10 bu.? 16 bu.? 20 bu.?

5. A man had 240 sheep and sold 48 of them; what per cent. did he sell?

6. A boy lost 15 papers of the 45 which he had bought; what per cent. did he lose?

7. $2\frac{1}{4}$ times a number is what per cent. of it?

8. In a school there are 100 boys and 150 girls; what per cent. more is the number of girls than of boys?

9. A farmer raised 500 bu. of oats and sold 200 bushels; what per cent. did he sell?

10. After 21 gallons had been drawn from a full hogshead of molasses, what per cent. remained?

11. What per cent. of $200 is $12\frac{1}{2}$% of $400?

12. A gold ring is 18 carats fine; what per cent. of it is gold, if pure gold is 24 carats fine?

13. What per cent. of a number is $\frac{5}{8}$ of it? $\frac{3}{4}$ of it? $\frac{7}{8}$ of it?

14. If wages were advanced from $40 to $45 a month, what was the rate of increase?

15. A man having a salary of $1000 a year spent $600; what per cent. did he save? what per cent. did he spend?

16. If I spend $2.35 out of $14.10, what per cent. do I spend? what per cent. do I save?

To Find the Base.

1. 40 is 5% of what number?

Solution.—If 5% or $\frac{5}{100}$ of a number is 40, $\frac{1}{100}$ is $\frac{1}{5}$ of 40 or 8, and $\frac{100}{100}$ or the number is 100 times 8 or 800. Therefore, 40 is 5% of 800.

Or, if 40 is 5 per cent. of a number, 1% is $\frac{1}{5}$ of 40 or 8, and 100% or the number is 100 times 8 or 800. Therefore, etc.

Or, since 40 is $\frac{1}{20}$ of some number, $\frac{20}{20}$ or the number is 20 times 40 or 800. Therefore, etc.

2. 25 is 5% of what number?

3. 40 is 8% of what number?

4. 10 is 10% of what number?

5. 75 is 25% of what number?

6. 33 is $33\frac{1}{3}$% of what number?

Solution.—If 33 is $33\frac{1}{3}$% or $\frac{1}{3}$ of a number, $\frac{3}{3}$ or the number is 3 times 33 or 99. Therefore, etc.

7. \$18 is $66\frac{2}{3}$% of how many dollars?

8. 60 bushels is $12\frac{1}{2}$% of how many bushels?

9. A boy sold a boat for \$12, which was 25% of the value of his bicycle; what was the value of his bicycle?

10. A horse was sold at a loss of \$50, which was 10% of the cost; what was the cost?

11. A's salary, \$800, is $66\frac{2}{3}$% of B's salary; what is B's salary?

12. A bookkeeper spent \$400 a year, which is 40% of his salary; what is his salary?

13. I spent \$50 for a dress; if the sum spent is 5% of all I had, how much money had I?

14. A farmer raised 150 bu. of barley, which was $12\frac{1}{2}$% of his crop of wheat; how much wheat did he raise, and what was its value at 75¢ per bu.?

A Number Increased or Diminished by a Certain Per Cent. of Itself.

1. What number increased by 20% of itself amounts to 30?

Solution.—If 30 is 20% or $\frac{1}{5}$ more than the number, 30 is 120% or $\frac{120}{100}$, equal to $\frac{6}{5}$ of the number; $\frac{1}{5}$ of the number is $\frac{1}{6}$ of 30 or 5, $\frac{5}{5}$ or the number is 5 times 5 or 25. Therefore, etc.

2. 45 is 25% more than what number?

3. A man sold a harness for $26, which was 30% more than it cost him; what did it cost him?

4. George gave 46¢ to a beggar, which was 15% more than James gave him; how much did the beggar receive from both?

5. Mr. Allen had 10 bbls. of potatoes, Mr. Smith had 4 times as many, which was 25% more than Mr. Stevens had; how many barrels did they all have?

6. Mr. Nimble walked 20 miles in a day and Mr. Fleet walked twice as far lacking 6 miles; the number of miles walked by both was 20% more than Mr. Racer walked; how far did Mr. Racer walk? how far did they all walk?

7. What number diminished by 40% of itself is 60?

Solution.—If 60 is 40% or $\frac{2}{5}$ less than the number, 60 is 60% or $\frac{60}{100}$, equal to $\frac{3}{5}$ of the number; 60 is $\frac{3}{5}$ of 5 times $\frac{1}{5}$ of 60 or 100. Therefore, etc.

8. What number diminished by 15% of itself is 51?

9. Having spent 25% of my money, I had 75¢ left; how much had I at first?

10. What number diminished by 20% of itself is 40?

11. A grocer sells tea at 90¢ per pound, which is 10% less than he paid for it; how much did he pay for it?

12. 68 yds. are 15% less than what number of yards?

13. A man sold a book for 60 cents and gained 25%; what did the book cost him?

SOLUTION.—Since he gained 25% or $\frac{1}{4}$ of the cost, $\frac{4}{4}$, the cost, plus $\frac{1}{4}$, the gain, or $\frac{5}{4}$ of the cost is the selling price or 60 cents. If $\frac{5}{4}$ of the cost is 60 cents, $\frac{1}{4}$ is $\frac{1}{5}$ of 60 cents or 12 cents, and $\frac{4}{4}$, or the cost, is 4 times 12 cents or 48 cents. Therefore, the book cost him 48 cents.

14. I sold a sewing machine for $46 and gained 15%; what did it cost me?

15. A grocer sold flour for $10 and gained 25%; what did the flour cost him?

16. A man having increased his bank deposit 20%, it amounted to $600; how much had he at first?

17. My income this year is $465, which is 7% less than it was last year; what was it last year?

18. A man sold 180 acres from his farm; if the number of acres sold was 10% less than the number of acres retained, how many acres did he retain? how many acres in his farm?

19. If a merchant make 8% by selling cloth at 27 cents a yard, what did it cost him?

20. Henry earned $90, which was 10% less than Samuel earned; how much did Samuel earn? how much did both earn?

21. What number increased by 40% of itself equals 49?

22. Sold a horse for $230, which was 15% more than his value; what was his value?

23. A boy spent 25% of his money and had 75 cents left; how much had he at first?

24. A grocer who sold coffee for $.45 a pound gained 12½% on its value; what was its value?

PROFIT AND LOSS.

PROFIT and LOSS are terms used to express the gain or loss in business transactions.

They may be computed by the principles of percentage.

TO FIND THE SELLING PRICE.

1. A merchant bought a piece of cloth for $80 and sold it at 25% profit; for how much did he sell it?

SOLUTION.—Since the gain was 25% or $\frac{1}{4}$ of the cost, the profit was $\frac{1}{4}$ of $80 or $20. $80, the cost, plus $20, the gain, equals $100, the selling price. Therefore, he sold it for $100.

2. A dealer bought hats at $5 apiece and sold them at 20% profit; what was the selling price?

3. At what price must flour, costing $6 a barrel, be sold to gain $16\frac{2}{3}$%?

4. Hats costing $5 apiece were sold at a loss of 20%; what was their selling price?

5. A grocer bought butter at 12 cents, 16 cents and 18 cents a pound; for how much must each kind be sold to gain $12\frac{1}{2}$%? to gain 25%?

6. A merchant sold velvet that cost $12 at an advance of 20%; what did he receive for it?

7. I sold tea at 15 cents above cost and gained $16\frac{2}{3}$%; what was the cost of the tea? what was the selling price?

8. A man bought a watch for $180 and sold it at a loss of $11\frac{1}{9}$%; what did he receive for it?

9. Mr. Jones paid $150 for a horse and sold him at $16\frac{2}{3}$% advance; what was his gain?

10. I bought a sewing machine for $40, spent $10 for repairs, and sold it at a gain of 25% on the whole cost; what did I sell it for?

11. A grocer bought tea at 75 cents a pound and sold it so as to gain 20%; what did he gain? what did he sell it for?

12. A lady bought a ring for $10 and sold it at a loss of 15%; what was the selling price?

To Find the Gain or Loss Per Cent.

1. A boy bought a pencil for 4 cents and sold it for 5 cents; what per cent. profit did he make?

Solution.—Since the gain on 4 cents is the difference between 5 cents and 4 cents, which is 1 cent or $\frac{1}{4}$ of the cost, the gain per cent. is $\frac{1}{4}$ of 100% or 25%. Therefore, he made 25% profit.

2. A girl bought a crochet needle for 10 cents and sold it for 8 cents; what per cent. did she lose?

3. A merchant bought cloth at $4 a yard and sold it at a gain of $2 a yard; what was his gain per cent?

4. I sold some butter at $\frac{1}{8}$ less than cost; what was the loss per cent.?

5. What per cent. is gained or lost by selling goods for $\frac{3}{4}$ of their cost? $\frac{2}{3}$? $\frac{4}{5}$? For double their cost? For $\frac{1}{2}$ the cost?

6. A shipper buys apples at $1.50 a barrel and sells them at a profit of $.50 a barrel; what per cent. does he gain?

7. Six times three dollars is what per cent. of nine times eight dollars?

8. An importer sold some damaged goods for $\frac{1}{16}$ of their cost; what was his loss per cent.?

9. A farmer sold a sheep for $12 and cleared $\frac{1}{4}$ of the amount of the sale; what would he have lost per cent. if he had sold it for $6?

10. What is the gain per cent. when $\frac{2}{3}$ of the article is sold for $\frac{3}{4}$ of its cost?

To Find the Cost.

1. James sold a pair of skates at 15% more than their cost and gained 18 cents; what was the cost?

Solution.—The gain, 15%, is $\frac{15}{100}$ or $\frac{3}{20}$ of the cost. If 18 cents is $\frac{3}{20}$ of the cost, $\frac{1}{20}$ of the cost is $\frac{1}{3}$ of 18 cents or 6 cents, $\frac{20}{20}$, or the cost, is 20 times 6 cents or $1.20. Therefore, the cost was $1.20.

2. A pound of coffee sold for 5 cents more than cost, and there was a gain of $16\frac{2}{3}$%; what was the cost?

3. A dealer sold a carriage for $9 less than cost, which was a loss of $16\frac{2}{3}$%; what was the cost?

4. A farmer sold wheat for 15 cents less a bushel than he had been offered for it, which was a loss of 20%; how much had he been offered for it?

5. I gave a boy 25 cents advance for a collection of postage stamps which he had just purchased; he afterwards said he had made a profit of 10%; how much did he pay for them?

6. The difference between $8 and $6 was one-half my loss when I sold cows at $12\frac{1}{2}$% below cost; how much did each cow cost me?

7. James sold his boat for $46, which was a gain of 15% on the cost; what was the cost?

Solution.—Since he gained 15% or $\frac{3}{20}$ of the cost, $46 is $\frac{20}{20}+\frac{3}{20}$ or $\frac{23}{20}$ of the cost. Since $46 is $\frac{23}{20}$, $\frac{1}{20}$ is $\frac{1}{23}$ of $46 or $2, and $\frac{20}{20}$, or the cost, is 20 times $2 or $40. Therefore, the cost was $40.

8. A man sold his horse for $51, which was a loss of 15%; what was the cost?

Solution.—Since 15% or $\frac{3}{20}$ was lost, he must have sold it for the difference between $\frac{20}{20}$ and $\frac{3}{20}$, or $\frac{17}{20}$ of the cost. If $51 is $\frac{17}{20}$ of the cost, $\frac{1}{20}$ of the cost is $\frac{1}{17}$ of $51 or $3. $\frac{20}{20}$, or the cost, is 20 times $3 or $60. Therefore, the cost was $60.

9. A furrier sold a set of mink furs at $12\frac{1}{2}\%$ less than cost and received $175. What was the cost?

10. A grocer lost 15% on some goods sold at 17¢ a pound; what was the cost per pound?

11. If a miller sells 4 barrels of flour at $5 a barrel, and on two he gains 25% and on two he loses 25%, does he gain or lose on both, and how much?

12. A grocer sold potatoes for 40 cents a bushel and lost 20%; what would he have received had he sold them for 20% gain?

Review Problems.

1. How shall I mark goods that cost $12 in order to gain $16\frac{2}{3}\%$?

2. How shall I mark goods that cost $5 so that I can deduct 10% from the marked price and yet make 8% on the cost?

Explanation.—First find the selling price by adding 8% or $\frac{2}{25}$ to $5, which is $5.40. $5.40 must be $\frac{9}{10}$ of the marked price.

3. A fish pole that was bought for $3 was sold for $2; what was the loss per cent.?

4. Find the profit on land that was bought for $200 and was sold at a gain of 12%.

5. What % is gained by selling apples at 20¢ a peck that cost 15¢ a peck?

6. A chair which cost $12 was sold for $9; what was the loss per cent.?

7. I sold a carriage for $55 and gained $37\frac{1}{2}\%$; how much did it cost?

8. How must I mark chairs that cost $6 so that I may deduct 10% from the marked price and still make 5% on the cost?

9. Find the per cent. gained on pecan nuts bought at 12¢ and sold at 14¢.

10. A boy bought a newspaper for 2¢ and sold it for 3¢; how much did he gain per cent.?

11. In a class of 40 pupils, 3 of every 5 failed in Arithmetic; what per cent. of the class failed and how many did not fail?

12. George had $62\frac{1}{2}$ cents and spent $\frac{3}{8}$ of a dollar for a knife; what per cent. of a dollar had he left?

13. A man failed in business and could pay but 40 per cent. of his debts; how much will a creditor receive on a bill of $200?

14. By selling ham at 7 cents a pound I gain $16\frac{2}{3}$%; what is the cost of the ham per pound?

15. An auctioneer bought a watch for $56, which was 20% less than its value; how must he sell it to gain 10% on its value?

16. Mr. C's house, having increased 30% in value, he sold it for $600 more than cost; what did it cost?

17. Edgar sold a book to Arthur for $1.90 and lost 5%; what did the book cost?

18. An article that cost $12 was sold so as to gain $3; what was the gain per cent.?

19. On an article that I sold for $20 I gained $2; what was my gain per cent.?

20. A music dealer sold two pianos at $240 each, one at a profit of 20% and the other at a loss of 20%; what was his entire gain or loss?

21. A dairyman, having a number of cows, bought 24 more, which purchase was 40% of the whole number he had; how many cows had he before he bought the last number?

TRADE DISCOUNT.

1. TRADE DISCOUNT is an allowance made by the seller to the buyer from the face of bills, list price of goods, etc., and is expressed by the term *per cent. off.*

Thus, 10% off, or 10 off, means an allowance of 10% from the asking price. 10 and 5% off means a discount, first of 10%, and then 5% from the remainder. Two fives and 10% off means two successive discounts of 5% and 10% from the remainder.

2. THE NET PRICE is the price received for the goods.

1. Bought a job lot of woolens for $200 on 3 mo. at a discount of 10%, and 5% off for cash; what was the net price?

NOTE.—This means that the goods were bought at the regular rates on the usual time at 10% off, but for cash a further discount of 5% from this would be made.

SOLUTION.—A discount of 10% on $200 is $\frac{1}{10}$ of $200, or $20 taken from $200. $200 — $20 = $180; 5% off for cash is $\frac{1}{20}$ of $180 or $9 taken from $180. $180 — $9 = $171, the net price. Therefore, etc.

2. What is the difference on a bill of $100 between a discount of 25% and a discount of 15 and 10 off?

3. What is the net cost of a bill of goods amounting to $80, bought on 4 mo. at 10% discount, and 5% off for cash?

4. Bought goods for $200 on 90 da. at 10% and 10%, and 5% off for cash; what is the discount?

5. I bought at a bankrupt sale a bill of shoes for $500 on 30 da. at 2 tens and 5% off for cash; what was the net price?

6. What is the net cost of a bill of goods for $100 on 2 mo. at 10, 10, and 5 off?

COMMISSION.

1. COMMISSION is a percentage paid to a person, called an agent, for the transaction of business for another.

2. All problems of commission may be solved by applying the principles of percentage.

3. The *money invested* is the base.

4. The *rate per cent.* allowed is the rate.

5. The *commission* is the percentage.

To FIND THE COMMISSION.

1. An agent sold $300 (base) worth of groceries and charged a commission of 5% (rate); how much did he receive (percentage)?

SOLUTION 1.—He received 5% or $\frac{5}{100}$ of $300. $\frac{1}{100}$ of $300 is $3, $\frac{5}{100}$ is 5 times $3 or $15. Therefore, etc.

SOLUTION 2.—He received 5% or .05 of $300 or $15. Therefore, etc.

2. An agent sold $1200 worth of pictures at a commission of 50%; how much did he receive?

3. A jeweler sold a diamond ring for $500 and received a commission of 10%; what was his commission?

4. An attorney collected bad debts to the amount of $800 and charged 20% commission; how much did he receive?

5. A druggist sold medicines on commission to the amount of $500; what did he receive at 40%?

6. An auctioneer sold goods at a commission of 8%; what did he receive from a sale of $300?

7. A salesman sold $1500 worth of dry goods; what was his commission at 6%?

8. An agent sold 50 bbls. of apples at $1.50 per bbl.; how much commission did he receive at 10%?

To Find the Rate of Commission.

1. A bookseller received $30 for selling $150 worth of books; what was the rate of commission?

Solution.—If he receives $30 for selling $150 worth of books, on $1 he will receive $\frac{1}{150}$ of $30 or $\frac{30}{150}$, which is $\frac{1}{5}$ or 20%. Therefore, etc.

2. A person received $6 for collecting a bill of $100; what was the rate of commission?

3. An agent received $15 on the sale of a sewing machine for $75; what was his rate of commission?

4. A nurseryman sold $240 worth of trees and received $80 commission; what rate % did he receive?

5. A tax collector received $27 for collecting $900; what is his rate of commission?

6. Find the rate of commission when an agent receives $5 for a sale of $200.

7. A city tax collector receives $18 for collecting $300 of overdue taxes; what is the rate?

8. An agent received $80 for selling a house worth $4000; what rate % did he receive?

9. I paid to my broker $25 for exchanging $1500 of foreign currency. What was the rate of brokerage?

10. Find the rate of commission when an agent receives $240 for collecting a debt of $8000.

11. A tax collector received $140 for collecting $7000; find the rate of commission.

12. An auctioneer sold goods amounting to $400, for which he charged $50 commission; what was the rate?

To Find the Amount of Sales.

1. A man received $40 for selling a horse at 10% commission; what did the horse cost?

Solution.—10% equals $\frac{10}{100}$ or $\frac{1}{10}$. If $40 is $\frac{1}{10}$ of the cost, $\frac{10}{10}$, or the whole cost, is 10 times $40 or $400. Therefore, etc.

2. An architect receives $150 on a 5% commission for the plans and general supervision of a building; what is the cost of the building?

3. Mr. King received $60 for collecting a note at 10% commission; what was the amount collected?

4. A music teacher received $50 for selling a piano at 20% commission; what was the value of the piano?

5. A dealer sold a lot of books and received $27 for his commission at 15%; what was the value of the books?

6. A huckster sold garden vegetables and received $3.30 commission at 15%; what was the amount of his sales?

7. 20% of $50 is the commission I received for collecting a note at 5%; what was the amount of the note?

8. 10% less than $10 was given me for selling two dozen steam cookers at 15% commission; what did each sell for?

9. Mr. French paid an agent $200 for selling a piece of property at 2%; what did the property bring?

10. An auctioneer received $60 for selling furniture at 3% commission; what was the value of the furniture sold?

11. A bookseller sold a lot of books on commission at 20%, and remitted $100 as net proceeds; for how much were the books sold?

12. A real estate broker received $40 for selling a house and lot at 5% commission; for how much was the property sold?

To Find the Amount Invested.

1. An agent receives $840 to invest in the purchase of shoes after deducting his commission of 5%; what amount does he invest?

Solution 1.—If the agent's commission is 5% or $\frac{1}{20}$ of the money invested, $840 is $\frac{20}{20}$ plus $\frac{1}{20}$, or $\frac{21}{20}$ of the money invested. $\frac{1}{20}$ is $\frac{1}{21}$ of $840 or $40; $\frac{20}{20}$, or the whole investment, is 20 times $40, or $800. Therefore, etc.

Solution 2.—The agent will receive $1.05 for every $1 he invests. He will invest as many dollars as $1.05 is contained times in $840, which is 800 times or $800. Therefore, etc.

2. An agent received $510; after deducting his commission of 2%, how much did he invest?

3. How much does an agent invest, if his employer sends him $412 to buy beef and allowed him 3% commission?

4. Mr. Quicktrade telegraphed for $520 with which to purchase small electric motors; how many can he buy at $5 each after deducting his commission of 4%?

5. A dry goods agent received $749 to invest in calicos after deducting his commission of 7%; how much did he invest?

6. Find the commission when an agent receives $336 to be invested in goods after deducting his commission at 12%.

7. An agent receives $108 with which to purchase peaches at 50¢ a basket after deducting his commission of 8%; how many baskets did he purchase?

8. A shoe dealer sent $990 to his agent in Rochester, N. Y., to be invested in ladies' shoes at $3 per pair, after deducting his commission of 10%; how much did he invest? How many pairs did he buy?

INSURANCE.

1. INSURANCE is security against loss.

2. THE POLICY is the contract between the insurer and the insured.

3. THE PREMIUM is the sum paid for the insurance.

1. A house was insured for $1500 at 1%; what was the premium?

SOLUTION.—The premium was 1% or $\frac{1}{100}$ of $1500, or $15. Therefore, etc.

2. A stock of goods was insured for $800 at 2%; what was the premium?

3. Mr. Rogers' cottage at the Lake is insured for $2000 for three years at 2%; what is the premium?

4. The premium for insuring a piano for $500 is $5; what is the rate of insurance?

SOLUTION.—If $5 is charged for insuring $500, it will require $\frac{1}{500}$ of $5 to insure $1 or $\frac{5}{500}$, equal to $\frac{1}{100}$ or 1%. Therefore, etc.

5. My house is insured for three years for $3000 and my furniture for $1000; if I pay $80 premium, what is the rate for three years?

6. A house valued at $6000 is insured for $\frac{2}{3}$ of its value at a cost of $40; what is the rate?

7. If it cost $60 to insure a certain house at $\frac{3}{4}$%, what is the amount of insurance?

8. It cost $60 to insure a store and its contents at 2%; what was the amount insured?

9. A man paid $60 premium for insuring a valuable horse at 5%; for how much was the policy written?

SOLUTION.—Since $\frac{3}{4}$% is $\frac{3}{400}$ of the amount insured, $60, the premium, is $\frac{3}{400}$ of the amount insured; $60 is $\frac{3}{400}$ of 400 times $\frac{1}{3}$ of $60. $\frac{1}{3}$ of $60 is $20, and 400 times $20 are $8000. Therefore, the amount of insurance is $8000.

INTEREST.

1. When one person lends another a sum of money, the person borrowing agrees to pay a certain per cent. of it each year for its use. The money thus paid is called *Interest.*

2. When the time expires during which the money was loaned, both the money borrowed and the interest are to be paid. This sum is called the *Amount.*

3. If money is loaned at 5% a year, what decimal part of the money borrowed equals the sum paid for its use?

EXPLANATION.—Since 5% equals .05, the interest for 1 year equals .05 of the principal.

4. At 5%, how many dollars must be paid for the use of $100 for 1 year? How many cents for 100 cents?

5. If $5 are paid for the use of $100 for 1 year, what is the rate per cent.?

6. INTEREST is money paid for the use of money.

7. PRINCIPAL is the sum for the use of which interest is paid.

8. AMOUNT is the sum of the principal and interest.

9. The RATE of INTEREST is the per cent. of the principal paid for its use *one* year.

10. The TIME is the period during which the money is on interest.

NOTE.—In computing interest, 30 days are usually considered a month, and 12 months a year.

TO FIND THE INTEREST.

1. Find the interest of $1 for 2 years at 5%.

SOLUTION 1.—The interest of $1 for 1 year at 5% is $.05 = \frac{5}{100}$, or $\frac{1}{20}$ of the principal, which is 5 cents. The interest for 2 years

is 2 times 5 cents, or 10 cents. Therefore, the interest of $1 for 2 years at 5% is 10 cents.

SOLUTION 2.—The interest of $1 for 1 year at 5% is 5¢, the interest for 2 years is 2 times 5 cents, or 10 cents. Therefore, etc.

2. Find the interest of $1 for 3 years at 6%.

3. Find the interest of $1 for 4 years at 7%.

4. Find the interest of $10 for 3 years at 6%.

SOLUTION.—The interest of $1 for 1 year at 6% is 6 cents, the interest of $10 for 1 year is 10 times 6 cents, or 60 cents; the interest of $10 for 3 years is 3 times 60 cents, or $1.80. Therefore, the interest of $10 for 3 years at 5% is $1.80.

5. What is the interest of $15 for 4 years at 4%?

6. What is the interest of $20 for $5\frac{1}{2}$ years at 5%?

7. What is the interest of $20 for 2 years 6 months ($2\frac{6}{12}$ or $2\frac{1}{2}$ years) at 10%?

8. I borrowed $50 and kept it for 3 years 4 mo.; how much interest did I pay at 6%? What was the whole amount paid at that time?

NOTE.—Amount equals the sum of principal and interest.

9. Mr. Try lent Mr. Cant $60 for 2 years at 5%; what amount did Mr. Cant owe Mr. Try at the expiration of the time?

10. What is the interest of $100 for 3 yrs. 8 mo. at 9%? (3 yrs. 8 mo. equal $3\frac{2}{3}$ years.)

SOLUTION.—The interest of $100 for 1 year at 9% is $9; for $3\frac{2}{3}$ years it is $3\frac{2}{3}$ times $9, or $33. Therefore, etc.

11. What is the interest of $200 for 2 yrs. 9 mo. at 8%?

12. What is the interest of $300 for 1 yr. 4 mo. at 7%?

13. What is the interest of $100 for 3 yrs. 2 mo. at 6%?

14. What is the amount of $40 for 1 yr. 6 mo. at 7%?

15. What is the interest and amount of $6 for 6 yrs. 6 mo. at 6%?

Six Per Cent. Method.

1. What is the interest of $1 at 6% for 1 yr.?

2 What is the interest of $1 at 6% for 1 mo.? For 2 mo.?

Solution.—The interest of $1 for 1 year at 6% is 6 cents, the interest for 1 mo. is $\frac{1}{12}$ of 6 cents, which is $\frac{1}{2}$ of a cent or 5 mills, and the interest for 2 mo. is 2 times 5 mills, which are 10 mills or 1 cent. Therefore, the interest of $1 at 6% for 2 mo. is 1 cent.

3. What is the interest of $1 at 6% for 4 mo.? 6 mo.?

4. What is the interest of $1 at 6% for 9 mo.? 10 mo.? 11 mo.?

5. What is the interest of $1 at 6% for 1 yr. 2 mo.? 2 yrs. 2 mo.? 3 yrs. 3 mo.?

6. What is the interest of $1 at 6% for 1 day? 6 days? 12 days?

Solution.—The interest of $1 dollar for 1 mo. or 30 days at 6% is 5 mills, the interest for 1 day is $\frac{1}{30}$ of 5 mills, which is $\frac{5}{30}$ or $\frac{1}{6}$ of 1 mill, the interest for 12 days is 12 times $\frac{1}{6}$ of 1 mill, or 2 mills. Therefore, the interest of $1 at 6% for 12 days is 2 mills.

7. What is the interest of $1 at 6% for 18 days? 24 days?

8. What is the interest of $1 at 6% for 9 days? 20 days? 25 days?

9. What is the interest of $1 at 6% for 17 days? 22 days?

10. What is the interest of $1 at 6% for 6 mo. 6 da.? 4 mo. 12 da.? 6 mo. 18 da.?

11. What is the interest of $1 at 6% for 1 year? 2 years? 3 years? 4 years?

12. What is the interest of $1 at 6% for 1 month? 2 months? 3 months? 4 months? 5 months? 6 months? 7 months? 8 months? 9 months?

13. What is the interest of $1 at 6% for 1 day? 2 days? 4 days? 6 days? 8 days?

14. What is the interest of $1 at 6% for 1 yr. 2. mo. 6 days?

15. What is the interest of $1 at 6% for 2 yrs. 4 mo. 12 days?

16. What is the interest of $1 at 6% for 1 yr. 6 mo. 18 da.?

17. What is the interest of $1 at 6% for 2 yrs. 8 mo. 21 da.?

18. What is the interest of $1 at 6% for 3 yrs. 6 mo. 6 da.?

The Interest of $1 at 6%

For 12 mo. or 1 year is 6 cents or .06 of the principal.

For 2 mo. or $\frac{1}{6}$ year is 1 cent or .01 of the principal.

For 1 mo. or 30 da. or $\frac{1}{12}$ year is 5 mills or .005 of the principal.

For 6 days or $\frac{1}{5}$ month is 1 mill or .001 of the principal.

For 1 da. or $\frac{1}{30}$ month is $\frac{1}{6}$ mill or .000$\frac{1}{6}$ of the principal.

From the foregoing analyses the following may be used to find the interest of $1 for the given time at 6%.

The interest of $1 for 1 year is 6 cents; for 1 month, $\frac{1}{2}$ cent; for 1 day, $\frac{1}{6}$ of a mill.

1. What is the interest of $1 for 2 yrs. 2 mo. 6 da. at 6%?

There will be —

Six times as many hundredths as there are years, - - - - -	$6 \times 2 = .12$
One-half as many hundredths as there are months, - - - - -	$\frac{1}{2}$ of $2 = .01$
One-sixth as many thousandths as there are days, - - - - -	$\frac{1}{6}$ of $6 = .001$
Interest on $1 for the given time at 6% =	$.131

To find the interest on any sum of money.

Multiply the interest of $1 for the given time, at 6%, by the principal.

If 7% interest be required, add $\frac{1}{6}$ of the interest at 6%. For 8%, add $\frac{1}{3}$, as 8% is $\frac{1}{3}$ more than 6%. For 9%, add $\frac{1}{2}$.

Thus, the interest of $1 for 2 yrs. 4 mo. 24 da. at 6% is 12 cents or .12, plus 2 cents or .02, plus 4 mills or .004.

Interest for 2 yrs.	=	$.12
Interest for 4 mo.	=	.02
Interest for 24 da.	=	.004
	6)	$.144 at 6%
		$.024 at 1%
	Adding,	$.168 at 7%

What is the interest of $1 at 6% for

1. 1 yr. 7 mo. 12 da.?	*6.* 4 yr. 8 mo. 16 da.?
2. 3 yr. 4 mo. 10 da.?	*7.* 8 yr. 3 mo. 3 da.?
3. 5 yr. 5 mo. 5 da.?	*8.* 6 yr. 11 mo. 10 da.?
4. 1 yr. 1 mo. 1 da.?	*9.* 5 yr. 6 mo. 7 da.?
5. 6 yr. 6 mo. 6 da.?	*10.* 10 yr. 10 mo. 10 da.?

11. What is the interest of $100 for 2 yr. 2 mo. 12 da. at 9%? What is the amount?

SOLUTION.—The interest of $1 for 2 yr. 2 mo. 12 da. at 6% is 13 cents, 2 mills ($.132); the interest of $100 is 100 times $.132, or $13.20. Since 9% is $\frac{1}{2}$ more than 6%, the interest of $100 at 9% will be the sum of $13.20 and $\frac{1}{2}$ of $13.20. $\frac{1}{2}$ of $13.20 is $6.60, which added to $13.20 equals $19.80. The amount is the sum of the principal, $100, and the interest, $19.80, or $119.80. Therefore, the amount of $100 for 2 yr. 2 mo. 12 da. at 9% is $119.80.

12. What is the interest of $300 for 3 yr. 4 mo. 12 da. at 5%?

13. What is the interest of $100 for 2 yr. 4 mo. 24 da. at 7%?

14. What is the amount of $100 for 4 yr. 4 mo. 9 da. at 12%?

NOTE.—12% is twice 6%, therefore, multiply the interest at 6% by 2 to get the interest at 12%. Then add the principal.

PROBLEMS IN INTEREST.

In *Problems in Interest* the three principal parts to be considered are: First, *The Principal;* second, *The Rate* per cent; third, *The Time.*

TO FIND THE PRINCIPAL.

1. What principal will in 6 yr. at 5% gain $60 interest?

SOLUTION.—$1 in 6 yr. at 5% will gain 30 cents; the principal that will gain $60 interest for the same time will be as many dollars as 30 cents or $\$\frac{3}{10}$ is contained times in $60, which is 200 times, or $200. Therefore, etc.

SUGGESTION.—$60, the interest, is $\frac{3}{10}$ of the principal.

2. What principal will in 2 yr. at 4% gain $16 interest?

3. What principal will in 5 yr. at 5% gain $20 interest?

4. What principal will in 6 mo. at 6% gain $15 interest?

5. What principal will gain $48 interest in 2 yr. at 6%?

6. What principal will gain $50 interest in 5 yr. at 5%?

7. What principal will gain $30 interest in 2 mo. at 6%?

8. What principal will gain $45 interest in 1 yr. 8 mo. at 9%?

9. What principal will gain $66 interest in 2 yr. 6 mo. at 12%?

10. What principal will amount to $21.80 in 1 yr. 6 mo. at 6%?

SOLUTION.—$1 in 1 yr. 6 mo. at 6% will *amount* to 109 cents or $\$\frac{109}{100}$; the principal that will amount to $21.80 will be as many dollars as 109 cents or $\$\frac{109}{100}$ is contained times in $21.80, which is 20 times or $20. Therefore, etc.

11. What principal will amount to $336 in 1 yr. 6 mo. at 8%?

12. What principal will amount to $412 in 6 mo. at 6%?

13. What principal will amount to $76.80 in 3 yr. 6 mo. at 8%?

14. What principal will amount to $555 in 1 yr. 10 mo. at 6%?

15. The amount due on a note which had been on interest for 3 years and 4 months at 9% is $520; what was the face of the note?

To Find the Time.

1. In what time will $200 at 6% gain $24 interest?

Solution.—Since $200 will gain $12 interest in 1 yr. at 6%, it will take as many years to gain $24 as $12 is contained times in $24 which is 2 times or 2 years. Therefore, etc.

2. In what time will $80 gain $9.60 interest at 3%?
3. In what time will $300 gain $42 interest at 7%?
4. In what time will $120 gain $40 interest at 10%?
5. In what time will $25 gain $7.50 interest at 6%?
6. In what time will $100 gain $32 interest at 4%?
7. In what time will $140 gain $28 interest at 5%?
8. In what time will $50 gain $10 interest at 8%?
9. In what time will $150 gain $30 interest at 6%?
10. In what time will $200 amount to $224 at 4%?

Suggestion.—$224 — $200 = $24 interest. Analysis as before.

11. In what time will $40 amount to $50 at 10%?
12. In what time will $100 amount to $142 at 6%?
13. In what time will $200 amount to $242 at 7%?
14. In what time will $80 amount to $120 at 5%?
15. In what time will $100 amount to $115 at 4%?

16. In what time will $10 amount to $20, or double itself, at 10%?

17. In what time will $15 amount to $30, or double itself, at 5%?

18. In what time will any principal double itself at any rate %?

NOTE.— Since the principal is to gain 100% of itself, it will take as many years as the rate % is contained times in 100%.

19. In what time will any principal double itself at 5%? 6%? 8%? 10%? 12%? 20%? 25%?

20. In what time will any principal treble itself at any rate %?

NOTE.— Since the principal is to gain 200% of itself, it will take as many years as the rate % is contained times in 200%.

21. In what time will any principal treble itself at 5%? 10%? 4%? 20%? 50%?

To Find the Rate.

1. At what per cent. will $60 gain $12 interest in 5 yr.?

SOLUTION 1.—The interest on $1 for 5 yr. at 1% is 5 cents, and on $60 it is 60 times 5 cents, which are 300 cents or $3; it will require as many per cent. as $3 is contained times in $12, or 4%. Therefore, etc.

SOLUTION 2.—At 1%, $60 in 5 yr. will gain $3 interest; it will take as many per cent. to gain $12 as $3 is contained times in $12, or 4%. Therefore, etc.

2. At what per cent. will $75 gain $15 interest in 2 yr.?

3. At what per cent. will $90 gain $13.50 interest in 3 yr.?

4. At what per cent. will $40 gain $3 interest in 2 yr. 6 mo.?

5. At what per cent. will $80 gain $8.40 interest in 1 yr. 9 mo.?

6. At what per cent. will $42 gain $9.10 interest in 2 yr. 2 mo.?

7. At what per cent. will $26 gain $4.16 interest in 2 yr.?

Review Problems.

1. Mr. Jones owes $250; what yearly interest does he pay at 8%?

2. Mr. Jones paid the above debt at the end of 3 years; what amount was due?

3. Mr. Stevens rents his house for $420, which he considers is 10% of its value; what is the value of the house?

4. What is the interest of $50 for 2 yr. 6 mo. at 6%?

5. I received $45 interest for a sum of money loaned for 2 yr. 6 mo. at 6%; what was the amount loaned?

6. A man pays a tax of $160 on his house, valued at $4000; what is the rate %?

7. In what time will $300 gain $42 interest at 7%?

8. What principal will gain $24 interest in 4 yr. at 6%?

9. What principal will amount to $530 in 18 mo. at 4%?

10. What principal will gain $36 interest in 3 yr. at 2%?

11. At what per cent. will $480 gain $72 interest in 3 yr.?

12. An agent sells 12 bbls. of potatoes at $2 per bbl.; what is his commission (percentage) at 5%?

13. Find the amount of sales (base) when a commission of 2% pays the agent $4.

14. When an agent receives $8 for collecting $400, what is the rate %?

15. In what time will any principal treble itself at 5%?

16. What is gained per cent. by selling coal at $6 a ton that cost $5 a ton?

17. Find the profit on land that cost $200 and was sold at a gain of 12%?

18. How must I sell goods that cost $56 so as to gain $37\frac{1}{2}$%?

19. My salary this year is $72 per month, or 20% more than last year; what was my salary last year?

20. After giving away 30% of my money, I found that I had $210 left; how much had I at first?

21. Mr. Trade sold his carriage for $91, which was at a loss of 35%; what should he have received for it to gain 40%?

22. A dealer sold two bicycles for $80 each; on one he lost 20% and on the other he gained 25%; did he gain or lose on both, and how much?

23. Mr. Williams sold two books for $15 each; on one he gained 25% and on the other he lost 25%; did he gain or lose on both, and how much?

24. $24 is 5% of the sum of C's and D's money; how much money has each, provided C has twice as much as D?

25. How many sheep at $4 each must a farmer buy, that, by selling them at a profit of 20%, he may gain $8?

Bankers' Short Methods for Finding Interest.

To find interest for days at 6%, 360-day basis, or common interest.

Explanation.—A principal of $1 will, in 1 year, at 6%, gain $.06 interest. A principal of $1 will, in $\frac{1}{6}$ year, or 2 months, or 60 days, at 6%, gain $.01 interest. Since $.01 equals $\frac{1}{100}$ of the principal, the interest on any sum of money for 60 days at 6% can be found by pointing off *two* integral places from the right, and,

since 6 is $\frac{1}{10}$ of 60, the interest for 6 days can be found by pointing off *three* places; and, since 10 times 60 is 600, the interest for 600 days is ten times that for 60 days, and can be found by pointing off *one* place; and, since 6000 is ten times 600, the interest for 6000 days can be found by multiplying the interest for 600 days by 10, or, in other words, the interest for 6000 days will equal the principal, the principal thus being shown to double itself in that time at 6%. This may further be proved true from either of two illustrations.

1st.—6000 da. ÷ 360 (12 × 30) = $16\frac{2}{3}$, or 16 yr. 8 mo.

2d.—100% ÷ 6% = $16\frac{2}{3}$, or 16 yr. 8 mo.

Hence, assuming $2136 as a principal, we form the following:

TABLE.

$2136 = principal.
$2.136 = interest at 6% for 6 days.
$21.36 = interest at 6% for 60 days.
$213.6 = interest at 6% for 600 days.
$2136. = interest at 6% for 6000 days.

NOTE.—Observe, as above stated, that the interest for 6000 days equals the principal.

1. To find the interest of any sum of money at 6% for 6 days.

Point off three integral places from the right of the principal.

2. To find the interest of any sum of money at 6% for 60 days.

Point off two integral places from the right of the principal.

3. To find the interest of any sum of money at 6% for 600 days.

Point off one integral place from the right of the principal.

4. To find the interest of any sum of money at 6% for 6000 days.

Write the interest as being equal to the principal.

To Find Interest at Other Rates than 6%, 360-Day Basis.

1. To find the interest on any sum of money for 12 days at 6%.

Point off 3 places and multiply by 2.

Suggestion.—1. For any number of days divisible by 6 proceed in like manner.

2. For other rates, add or subtract fractional parts of results.

3. For odd days, add fractional parts to the result.

2. To find the interest for 24 days at 5%.

Point off 3 places, multiply by 4, and from the result subtract one-sixth of itself.

3. To find the interest for 10 days at 6%.

Point off 2 places and divide the result by 6.

4. To find the interest for 20 days at 7%.

Point off 2 places, divide the result by 3, and to the quotient add one-sixth of itself.

5. To find the interest for 45 days at 8%.

Point off 2 places, subtract from the result one-fourth of itself, and to the remainder add one-third of itself.

6. To find the interest for 50 days at 6%.

Point off 1 place and divide by 12.

7. To find the interest for 40 days at 6%.

Point off 2 places, and from the result subtract one-third of itself.

Suggestion.—Daily class drill for five or ten minutes, during the time given to the subject of interest, will impart to the class an astonishing degree of accuracy and rapidity in computing interest.

Examples for Practice.

1. Find the interest on $150 for 30 days at 6%.

Solution.—Point off 2 integral places from the right of the principal, which gives $1.50, the interest for 60 days; divide the result by 2, which gives $.75, or the required interest for 30 days. Therefore, etc.

2. Find the interest on $3.30 for 40 days at 6%.

Solution.—Point off 2 integral places from the right of the principal, which gives $.033, the interest for 60 days; subtract from this result one-third of itself, as 40 is one-third less than 60, and we have $.022, the required interest for 40 days. Therefore, etc.

3. Find the interest on $180 for 90 days at 6%.
4. Find the interest on $120 for 80 days at 6%.
5. Find the interest on $360 for 100 days at 6%.
6. Find the interest on $42 for 300 days at 6%.
7. Find the interest on $400 for 75 days at 6%.
8. Find the interest on $16.80 for 30 days at 6%.
9. Find the interest on $350 for 120 days at 8%.

Explanation.—Point off 2 places (60 days), multiply the result by 2 (120 days), and to the product add one-third of itself (8%).

10. Find the interest on $28 for 45 days at 8%.
11. Find the interest on $500 for 12 days at 9%.
12. Find the interest on $60 for 60 days at 10%.
13. Find the interest on $90 for 9 days at 6%.
14. Find the interest on $480 for 24 days at 6%.
15. Find the interest on $600 for 66 days at 7%.
16. Find the interest on $20 for 18 days at 3%.

Explanation.—Point off 3 places (6 days), multiply the result by 3 (18 days), and divide the result by 2 (3% is ½ of 6%).

17. Find the interest on $60 for 40 days at 8%.
18. Find the interest on $90 for 120 days at 6%.

19. Find the interest on $120 for 100 days at 9%.

EXPLANATION.—Point off 1 place (600 days), divide by 6 (100 days), and to the quotient add one-half of itself (interest at 9%).

Or, point off two places (60 days), to the result add two-thirds of itself (100 days), and to the sum add one-half of itself (9%).

20. Find the interest on $800 for 66 days at 3%.
21. Find the amount of $40 for 35 days at 6%.
22. Find the amount of $660 for 60 days at 8%.
23. Find the amount of $500 for 120 days at 9%.
24. Find the amount of $36.42 for 60 days at 6%.
25. Find the amount of $300 for 15 days at 4%.

1.

GENERAL REVIEW PROBLEMS.

1. A boy, having a certain number of apples, bought 6 and sold 10, and then had none left; how many had he at first?

2. George had 11 cents, Arthur gave him 12, and Dick gave him enough to make his number 30; how many did Dick give him?

3. A grocer, having 10 melons, sold 6, and then bought 5 times as many as he sold; how many had he then?

4. Two bicycle riders start from the same place and travel in opposite directions, one at the rate of 8 miles an hour and the other at the rate of 9 miles an hour; how far will they be apart in 5 hours?

5. Two bicycle riders start from the same place and travel in the same direction, one at the rate of 6 miles an hour, the other at the rate of 10 miles an hour; how far apart will they be in 7 hours?

6. A boy gave 7 pencils, worth 4 cents each, for 3 packages of envelopes, worth 8 cents a package; what did he lose?

7. Eight times a number, divided by 4, multiplied by 6, divided by 3, multiplied by 5, divided by 10, equals how many times the number?

8. Think of a number, multiply it by 10, divide by 5, multiply by 3, divide by 6, add 16, subtract the number thought of, divide by 8, and what is the quotient?

9. Three times a number, minus 2 times the number, plus 4 times the number, plus 2 times the number, equals how many times the number?

10. A farmer, having 40 sheep, lost 20 and found only $\frac{3}{5}$ of them; how many had he then?

11. An agent, having 40 barrels of flour, sold $\frac{3}{4}$ of them, and then bought $\frac{1}{3}$ as many as he sold; how many had he then?

12. Mr. Rice, having $140, gave $\frac{3}{7}$ of it to the poor and lost $\frac{3}{4}$ of the remainder; how much then remained?

13. If I exchange 12 oranges for apples at the rate of 3 oranges for 9 apples, how many apples should I receive?

14. Mr. Quickstep walked 5 hours at the rate of 12 miles in 3 hours; how far did he travel?

15. If 7 yards of cashmere cost $21, what will $\frac{2}{3}$ of 15 yards cost?

16. A grain merchant gave 12 bushels of corn for 2 barrels of flour; what was the corn worth a bushel if 8 barrels of flour cost $48?

17. If 5 boys can do a piece of work in 16 days, how long will it take 20 boys to do it?

18. A grocer bought 7 dozen eggs at the rate of 8 cents for $\frac{2}{3}$ of a dozen; how much did they cost?

19. A merchant, having 25 baskets of peaches, sold $\frac{3}{5}$ of them to A and $\frac{3}{5}$ of the remainder to B; how many baskets remained?

20. Think of a number, multiply by 12, divide by 6, multiply by 2, divide by 4, add 12, subtract the number thought of, divide by 4, and name the result.

2.

1. If $\frac{2}{3}$ of 9 apples cost $4\frac{4}{5}$ cents, what will 15 apples cost?

2. What cost $\frac{1}{2}$ of 8 yards of cloth if $\frac{1}{2}$ of 6 yards cost $\frac{1}{3}$ of $27?

3. 5 is $\frac{1}{7}$ of what number? $\frac{1}{12}$ of what number?

4. 9 is $\frac{1}{2}$ of 6 times what number?

5. The head of a fish is 3 inches long and the tail 4 inches, which is $\frac{1}{2}$ of $\frac{1}{3}$ of the length of the body; required, the length of the fish.

6. A marketman has 24 geese, and $\frac{3}{4}$ of the number of geese equals 2 times the number of turkeys; how many turkeys has he?

7. If 4 men can do a piece of work in 18 days, how long will it require 12 men to do it?

8. Charles gave his sister 20 cents, which is $\frac{4}{5}$ of what he had at first and $\frac{1}{2}$ of what his sister now has; how much had each at first?

9. Mr. Dress gave $24 for a watch, and $\frac{1}{2}$ plus $\frac{2}{3}$ of this is 4 times what he paid for a chain; what did he pay for the chain?

10. A farmer who had $56\frac{3}{4}$ acres of land, sold $\frac{2}{3}$ of it; how many acres did he sell?

11. One-half of A's age was $\frac{4}{5}$ of B's; how many times B's age was A's?

12. $\frac{5}{8}$ multiplied by another number gives $\frac{3}{5}$; what is the multiplier?

13. What number is that to which if its $\frac{1}{3}$ be added the sum will be 56?

14. Three times a certain number increased by $\frac{3}{4}$ of itself equals 75; what is the number?

15. What number is that which, being increased by the difference between its $\frac{1}{4}$ and $\frac{1}{5}$, equals 63?

16. Charles, having $\frac{2}{5}$ of a dollar, gave $\frac{1}{2}$ of it to James, and James gave $\frac{1}{4}$ of his to John; what part of a dollar did each then have?

17. Mr. Farmer has 60 fruit trees, $\frac{2}{5}$ of which are peach trees, $\frac{2}{3}$ of the remainder are pear trees, and the remainder are apple trees; how many are there of each?

18. If $\frac{2}{3}$ of a number increased by 8 equals 32, what is the number?

19. A fishing rod is 15 feet long, and $\frac{2}{5}$ of its length lacks 3 feet of being $\frac{3}{4}$ of the length of the line; what is the length of the line?

20. Divide the number 48 into two such parts that twice the first part diminished by 6 shall equal the second part.

3.

1. Twice the sum of two numbers is 30, and 3 times the smaller equals 2 times the greater; what are the numbers?

2. Three men bought 75 horses, and as often as the first paid $4 the second paid $5 and the third $6; how many horses should each receive?

3. Divide the number 49 into two parts which are to each other as $\frac{1}{3}$ to $\frac{1}{4}$.

4. If 7 men can earn $28 in 4 days, how many dollars can 9 men earn in 6 days?

5. If it require 5 men 8 days to build 20 rods of wall, how many men can in 2 days build $\frac{1}{2}$ as much wall?

6. If 4 men can do a piece of work in 6 days, in what time will it be completed if they receive the assistance of 5 men when the work is half done?

7. The difference between $\frac{1}{2}$ of a number and $\frac{1}{4}$ of it is 5; what is the number?

8. What part of 1 yard is 2 feet?

9. How many square yards in 108 square feet?

10. How do you find the area of a square or rectangular surface?

11. How many sq. yds. in the floor of a room that is 18 ft. by 20 ft.?

12. If the ceiling of the above room were 10 ft. high, how many square feet in one end of the room?

13. How do you find the cubic contents of anything?

14. How many cu. ft. in a cistern that is 5 feet long, 4 feet wide and 8 feet deep?

15. Name the months which have 30 days each.

16. Divide the number 50 into two parts that shall be to each other as 7 to 3.

17. If 6 sheep are worth 2 cows, and 10 cows are worth 5 horses, how many sheep can you buy for 3 horses?

18. If 8 boys can do a piece of work in 5 hours, in what time will the work be completed provided 3 boys leave when the work is half done?

19. At 20 cents a sq. ft., what will be the cost of a piece of land 20 feet by 40 feet?

20. How many yards of carpeting, 1 yard wide, will be required to cover a hall floor 6 ft. wide and 15 feet long?

4.

1. A, B and C can build a wall in 5 days; B and C can do it in 8 days. In what time can A do it alone?

2. What will 1 mile of telegraph wire cost at 5 cents a foot?

3. A building lot containing 4000 sq. ft. is 100 feet long; how wide is it?

4. If a ton of hay cost $20, how much is it a pound?

5. If I buy 8 pounds of sugar of you at $6\frac{1}{2}$ cents a pound and give you $1, state how you will make the change?

6. What must I pay for a piece of land 20 rods by 40 rods at the rate of $50 an acre?

7. How many single things in $\frac{1}{2}$ of a gross?

8. What part of an acre is a lot that is 6 rods front and 10 rods deep?

9. If 1 bushel of walnuts cost $1.60, what will be the cost of 6 quarts?

10. How many days will it take 5 men to earn $\$11\frac{2}{3}$ if 3 men earn $\$\frac{7}{8}$ in $\frac{1}{4}$ of a day?

11. Bought a barrel of vinegar for $4 and sold it at 5 cents a quart; how much was gained?

12. If 3 furnaces consume $12\frac{1}{4}$ tons of coal in 7 days, how long will $17\frac{1}{2}$ tons last 5 furnaces?

13. The difference between two numbers is 8, and $\frac{1}{11}$ of the larger is equal to $\frac{1}{7}$ of the smaller; what are the numbers?

14. A tree, by falling, was broken into three pieces; the top part was 15 ft. long, the bottom part as long as the top and $\frac{1}{2}$ of the middle piece, and the middle piece as long as the top and bottom; what was the length of the tree?

15. I sold my horse for $128, which was $\frac{4}{7}$ of what he cost; how much did I lose on him?

16. If 6 pounds of cheese cost .9 of a dollar, what will 1 pound cost?

17. From 4.5 take .25; from 2 take .005.

18. At .6 of a dollar per yard, what will 1.2 yards of cloth cost?

19. Miss Kitchen paid .35 of a dollar for sugar, .7 for tea, .9 for coffee; how much did she pay for all?

20. What is the sum of .3, .5, .08 and .125?

5.

1. $\frac{8}{9}$ is $\frac{5}{8}$ of what number? $\frac{3}{4}$ is $\frac{6}{7}$ of what number?

2. Divide .25 by .05; .2 by .04; .6 by .012.

3. $\frac{2}{11}$ of 121 are $\frac{2}{9}$ of how many times 7?

4. Six times a number is 15 greater than three times the same number; what is the number?

5. Two men bought 48 bushels of apples; one paid $7 and the other $5; how many bushels should each have?

6. How many hundredths of a number is 15 per cent. of it?

7. What fractional part of a number is $12\frac{1}{2}$% of it?

8. If I have $600 in a bank and draw out $16\frac{2}{3}$%, how much remains?

9. If a man spend $\$1\frac{5}{6}$ a month for tobacco, in what time will he spend $\$16\frac{1}{2}$ at the same rate?

10. A grocer bought 10 baskets of eggs, containing 20 dozen each, but found, on unpacking them, 8% broken; how many dozen remained?

11. A man bought a house for $4000 and was obliged, during a period of "hard times," to sell it for $3000; what per cent. did he lose?

12. If a clerk spend $\frac{4}{5}$ of his salary, what per cent. of it does he save?

13. A farmer, having 160 sheep, sold all but 20% of them; what part did he sell, and what % did he sell?

14. What part of 48 miles is 6 miles?

15. From a hogshead of wine 42 gallons were drawn; what per cent. of the whole remained?

16. A quantity of produce was sold for $80, at a sacrifice of $\frac{3}{7}$ of the cost; what was the loss on it?

17. A man pays $6 an acre for the use of land valued at $75 an acre; what per cent. of the value of the land does he pay?

18. $40 is 8% of what sum of money?

19. A farmer, having 200 barrels of potatoes, sold $\frac{1}{4}$ of them to one man and $\frac{1}{3}$ of the remainder to another; what % remained unsold?

20. I loaned a man a sum of money; at one time he paid me $80, which was 16% of the whole sum I loaned him; how much did I loan him?

6.

1. A man, owning 30% of a mill, sold $33\frac{1}{3}$% of his share for $1500; what was the value of the mill?

2. I gave Henry $\frac{4}{5}$ of all the money I had; if he spent $\frac{3}{4}$ of what I gave him, and had $10 left, how much money did I give him?

3. 250 is 25% more than what sum of money?

4. A man is 64 years old; $12\frac{1}{2}$% of his age is $33\frac{1}{3}$% of the age of his oldest son; how old is his son?

5. A farm cost $3000; $\frac{2}{3}$ of this sum was $\frac{5}{8}$ of 4 times the cost of the barn that was on it; what was the cost of the barn?

6. An auctioneer sold goods to the amount of $400; what does his commission amount to at $2\frac{1}{2}$%?

7. A man, failing in business, can pay but 40% of his debts; how much will a creditor receive on a debt of $225?

8. A collector received $60 for collecting a bill at 4% commission; how much money did he collect?

9. An agent sells my house and lot for $4000 and charges me $1\frac{1}{2}$% commission; how much does he receive?

10. A man divided $100 among his 2 sons and 3 daughters, giving each son $\frac{1}{2}$ as much as each daughter; how much did each receive?

11. At $4\frac{1}{2}$%, what amount of insurance can be obtained for $90 on a car-load of stock?

12. Three persons hire a pasture for $50; one put in 25 sheep, another 35 sheep, and another 10 cows; one cow being equal to 4 sheep, what should each pay?

13. When stock is at 5% premium, what is the market value of $1?

SUGGESTION.—When stock is sold for its original or face value, it is said to be *at par*, represented by 100%. When stock is sold for more than its face value, it is said to be *above par* or *at a premium*. When less than its face value, *below par* or *at a discount*. Thus, 100 denotes *at par*; 105, 5% *above par*; 95, 5% *below par*.

14. When stock is at 10% discount, what is the market value of $1?

15. How much will 10 shares ($100 each) of railroad stock cost at 8% premium?

16. Paid $12 premium, at $1\frac{1}{2}$%, for insuring a boat-load of wheat; what was the amount of insurance?

17. How much will 5 shares of bank stock cost when quoted at 95 (par value $100)?

18. If an insurance company declare a dividend of 4%, how much will a stockholder receive who owns 20 shares?

19. A ferry company made an assessment of $3\frac{1}{2}$% for repairs; how much did a stockholder pay who owned 12 shares?

20. How much bank stock can be bought for $920 when quoted at 115? at 92?

7.

1. Paid $12 for 15 bushels of oats; what must they be sold for per bushel to gain 25%?

2. A man bought 60 barrels of apples for $180 and sold them at a loss of 20% on the cost; how much did he receive per barrel for them?

3. I bought a building lot for $200 and sold it at an an advance of 225%; how much was my gain?

4. Mr. McCabe bought a carriage for 20% less than $150 and sold it for 10% more than $150; what was the gain per cent.?

5. What per cent. is lost by buying potatoes at 50¢ a bushel and selling them at 40¢?

6. What is the annual interest of $150 at 6%?

7. What is the yearly interest of $75 at 4%?

8. What is the interest of $50 for 2 years at 5%?

9. What is the interest of $150 for 5 years at 8%?

10. A man pays $600 rent, and $\frac{3}{4}$ of this is just $\frac{1}{3}$ of $\frac{1}{2}$ of twice his income; what is his annual income?

11. I sold flour at $12 a barrel that I bought at $\frac{1}{6}$ off from the same price; what per cent. did I make?

12. Sold a bill of goods at wholesale at 40% off, and 5% off for cash; what was the total discount?

13. A grocer bought tea that he could sell for $.90 a pound and make $33\frac{1}{3}$% profit; how much did it cost him?

14. What is the interest of $200 for 1 year 10 months at 6%?

15. What is the interest of $300 for 9 months at 8%?

16. A speculator sold barley at $.90 a bushel, losing 10%; at what price should he have sold it to gain 15%?

17. I sold a house and lot for $\frac{3}{4}$ of $1\frac{3}{5}$ times its cost; what was the gain %?

18. What is the amount due on a note of $500 on interest 1 year 4 months at 6%?

19. In what time will $200 double itself at 5%?

20. How shall a merchant tailor mark goods that cost $8 so as to fall 20% from the marked price and yet make 25% on them?

8.

1. What per cent. is gained by buying oil at 80 cents a gallon and selling it at 12 cents a pint?

2. If I sell goods for twice the cost, what % do I gain?

3. If I buy an article for $10 and sell it at $4, what per cent. do I lose?

4. A's money is 50% more than B's; how many per cent. less is B's than A's?

5. At the rate of 10%, what commission does an agent receive for collecting a note of $800?

6. An attorney was paid $3\frac{1}{2}$% to collect a debt of $240; how much did he receive for his services?

7. A merchant sends his agent $780 to invest in merchandise after deducting his commission of 4%; what sum will he expend?

8. At 3%, what amount of insurance can be obtained on a house for $75 premium?

9. A grocer sold a quantity of butter for $24, by which he gained 20% on the cost; what was the cost?

10. A and B have $70; $\frac{1}{3}$ of A's money equals $\frac{1}{4}$ of B's; how many dollars has each?

11. Find the interest of $500 for 10 months at 6%.

12. Find the amount of $80 for 2 years 6 months at 8%.

13. How much does a man owe who borrowed $200 and kept it for 3 years 4 months 24 days at 6%?

14. $\frac{4}{3}$ of 36 is $\frac{4}{9}$ of 9 times $\frac{2}{7}$ of what number?

15. $\frac{4}{9}$ of 9 times $\frac{2}{7}$ of 42 is 4 times what part of 36?

16. A market-woman bought 30 oranges at the rate of 2 for a cent, and as many lemons at the rate of 3 for a cent, and sold the whole at the rate of 5 for 2 cents; did she gain or lose, and at what rate per cent.?

17. What is the amount of $260 for 4 yr. 4 mo. 24 da. at 6%?

18. Divide $375 between A and B so that A will receive $35 more than B. How much will each receive?

19. At what price should damaged goods be marked to lose 25%, the first cost having been 36¢ per yard?

20. After a carriage had been used two years it was sold for $5 less than it cost, the seller sustaining a loss of $3\frac{1}{3}$% of the cost; how much was the first cost of the carriage?

METRIC SYSTEM.

1. The METRIC SYSTEM of weights and measures originated in France in 1795, and is a decimal system of denominate numbers. It has been adopted by the leading nations of the world.

2. The METER, *the unit of length*, is the base of the system, and is one ten-millionth part of the distance on the earth's surface from the equator to either pole, or 39.37079 inches.

NOTE.—The term *meter* is derived from the Greek *mĕtron*, a *measure*.

3. The principal units of the system are:

The Meter, the unit of length,	39.37 inches.
The Are (air), the unit of surface	119.6 square yards.
The Liter (leeter), the unit of capacity	1.0567 qts., wine measure. .908 qt., dry measure.
The Gram (grăm), the unit of weight	15.432 grains.
The Stere (stāre) the unit of solids or volumes	$\frac{1}{4}$ cord (nearly).

4. The *multiple* units, or higher denominations, are expressed by prefixing to the standard or primary unit the numerals, Deka (10), Hekto (100), Kilo (1000), Myria (10000).

Thus, Dekameter means 10 meters; Kiloliters, 1000 liters, etc.

5. The *sub-multiple* units, or lower denominations, are expressed by prefixing to the primary units the numerals, Deci ($\frac{1}{10}$), Centi ($\frac{1}{100}$), Milli ($\frac{1}{1000}$).

MEASURE OF LENGTH.

The Meter is the *unit of length,* which is equal to 39.37 inches nearly. The divisions are the decimeter, centimeter and millimeter; the multiples are the dekameter, hektometer, kilometer and myriameter.

TABLE.

10 mil'li-me'ters(mm.)	=1 cen'ti-me'ter..cm.	$=\frac{1}{100}$ M.
10 cen'ti-me'ters	=1 dec'i-me'ter ..dm.	$=\frac{1}{10}$ M.
10 dec'i-me'ters	=1 Meter .....M.	
10 me'ters	=1 dek'a-me'ter..Dm.	=10 M.
10 dek'a-me'ters	=1 hek'to-me'ter.Hm.	=100 M.
10 hek'to-me'ters	=1 kil'o-me'ter ..Km.	=1000 M.
10 kil'o-me'ters	=1 myr'ia-me'ter.Mm.	=10000 M.

All tables are formed in a similar manner.

Metric Weights and Measures, like U. S. Money, are written as whole numbers and decimals of the denomination which is taken for the *unit,* as in the following

TABLE.

Tens of Thou.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.
5	4	3	2	1	.	1	2	3
Mm.	Km.	Hm.	Dm.	M.		dm.	cm.	mm.

The above is read the same as a whole number and decimal, 54321.123 meters. Expressed in centimeters it is 5432112.3 centimeters; in kilometers it is 54.321123 kilometers.

To express Metric Weights and Measures decimally in terms of any denomination,

Write the several denominations one after another in their order, and place a decimal point between the given denominator and those below it. If any intervening denominations are omitted in the given number, their places must be supplied with ciphers.

To reduce higher Metric Denominations to lower.

Multiply the given denomination by 10, 100, 1000, etc., as the case may require, and point off the product as in multiplication of decimals.

To reduce lower Metric Denominations to higher.

Divide the given denomination by 10, 100, 1000, etc., as the case may require, and point off the quotient as in division of decimals.

EXAMPLES.

1. In 1 kilometer, how many meters?
2. In 1000 millimeters, how many meters?
3. How many steres in 18 dekasteres of wood?
4. How many deciliters in 24½ liters of wine?
5. How many meters in 472 centimeters?
6. How many millimeters in 472 meters?
7. In 10000 centimeters, how many meters?
8. Reduce 1658 dekameters to meters.
9. Reduce 216 grams to decigrams.
10. Reduce 3215 decigrams to dekagrams.
11. Reduce 12.56 liters to centiliters.
12. Reduce 9876.5 grams to kilograms.
13. Reduce 8754 decimeters to millimeters.
14. Reduce 312,789 meters to hektometers.
15. Reduce 67.89 kilometers to meters.